

Advanced Number Line Concepts





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Common Core Standards

2.MD.B.6:

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

6.NS.C.6.A:

Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

6.NS.C.6:

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

1.MD.B.3:

Tell and write time in hours and half-hours using analog and digital clocks.

4.NF.C.7:

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.

3.NF.A.2:

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

4.NF.C.6:

Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

6.NS.C.5:

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.C.7.C:

Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.

6.NS.C.6.B:

Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

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Introduction

The number line, a commonplace feature in elementary and middle school classrooms, is often underestimated in terms of its pedagogical potential. It is usually hung above the blackboard, making it easily noticeable but unfortunately not optimally used. Teachers typically employ it for introducing young learners to ordinal numbers and helping them get a grip on counting.

Occasionally, it serves a role akin to a ruler, demonstrating benchmark fractions like $\frac{1}{2}$ or $\frac{1}{4}$. However, its application rarely extends beyond showcasing elementary whole numbers and a few fractions. This situation belies the true potential of the number line as a mathematical tool that can nurture a robust understanding of number relationships and encourage the development of operation skills among students.

Lately, the significance of the number line in cultivating mental agility in arithmetic and promoting a deeper comprehension of mathematical concepts has gained traction in educational research. This shift has been fueled, in part, by concerning performances by students on upper elementary-grade arithmetic problems.

One eye-opening study from the Netherlands, a country celebrated for its strong mathematical education, unveiled that just half of the student cohort could correctly solve a problem like $64-28$. Moreover, an even smaller fraction demonstrated adaptability in applying arithmetic strategies. Such findings have urged educators to reconsider traditional models for fostering basic number sense and computational fluency.

Contrary to popular belief, recent research findings suggest that the conventional tools and models utilized for teaching arithmetic relationships and operations may not be as beneficial as initially assumed. Base-10 blocks, while useful for conceptual comprehension, were found to provide insufficient procedural representation of numerical operations.

The hundreds chart, considered a step up from arithmetic blocks, nevertheless had its shortcomings. It was deemed overly intricate for many students who had difficulty mastering its use. In contrast, the number line presents an intuitive model, providing substantial advantages in assisting students to grasp the relative size and position of numbers and to visually interpret operations.

In the 90s, mathematicians from the Netherlands pioneered the re-emergence of the "empty number line," revitalizing this long-standing model as potentially the most significant tool within the sphere of number and operation. Since then, educators globally have embraced this superb model, yielding impressive outcomes.

Select Lessons

Main Concepts

What is a Number line

Imagine a really long straight line, stretching both to your left and right, far beyond what your eyes can see. Now, imagine that this line is like a ruler, but instead of measuring inches or centimeters, it's used to measure numbers. This, my friends, is a number line!

The number line is like a map for numbers. Just like on a treasure map where "X" marks the spot, each point on the number line represents a specific number. At the center of our number line, we have the number 0. If we move to the right, the numbers get bigger (1, 2, 3, and so on). If we move to the left, we start to encounter negative numbers (-1, -2, -3, and so on).

How to use a number line:

Number lines are really handy tools in math! They help us understand where numbers belong in relation to each other. Here are some ways we can use them:

Understanding Order of Numbers: On a number line, numbers to the right are always larger and numbers to the left are always smaller. If you have the numbers 3 and 5, you can tell that 5 is bigger because it's to the right of 3 on the number line.

Adding and Subtracting: You can also use a number line to add and subtract numbers. If you're at number 3 and you need to add 2, you just take two steps to the right to land on 5. To subtract, you move to the left. So, if you're at number 5 and need to subtract 3, you take three steps to the left to reach 2.

Identifying Negative Numbers: Negative numbers can seem a bit tricky, but the number line makes them easier to understand. Negative numbers are to the left of 0 on the number line. They get smaller as you move further left. So -1 is bigger than -2, even though 1 is usually smaller than 2.

Understanding Fractions and Decimals: Number lines can also help us understand fractions and decimals. Between any two numbers, like 0 and 1, there are lots of other points that represent fractions and decimals. For instance, halfway between 0 and 1, we have the point 0.5, or $\frac{1}{2}$.

Lesson 1

How to place numbers on a Number Line.

Today, we're going to learn how to place numbers on our number line. Get ready to be a number line expert!

Step 1: Draw Your Number Line

First, grab a piece of paper and a pencil. Draw a straight, horizontal line. This is your number line. It represents all the numbers that exist, from the smallest negative number to the largest positive number. That's a lot of numbers!

Step 2: Mark the Middle Point

Now, let's find the center point on your line and make a small vertical mark. This is where the number zero (0) lives. Above or below this mark, write a "0." Great job! You've just placed your first number on the number line.

Step 3: Adding Positive Numbers

To the right of the zero, make more small vertical marks. Each of these marks represents a positive number. Above or below each mark, write the number it represents. Start with 1 for the first mark to the right of 0, then 2 for the second mark, and so on. You've just placed positive numbers on the number line!

Step 4: Adding Negative Numbers

The left side of zero is where the negative numbers live. Just like you did with the positive numbers, make small vertical marks to the left of the zero. These marks represent negative numbers. Start with -1 for the first mark to the left of 0, then -2 for the second mark, and so on. Fantastic! Now you have negative numbers on your number line!

Step 5: Placing Other Numbers

What if you have to place a number that's not a whole number, like a fraction or a decimal? Easy-peasy! Let's say you need to place $\frac{1}{2}$ or 0.5 on your number line. This number lives halfway between 0 and 1. So, find the space between 0 and 1 and make a mark in the middle. Write " $\frac{1}{2}$ " or "0.5" above or below it. Awesome! You've just placed a fraction and a decimal on your number line!

Remember: The number line is all about order. Positive numbers are always to the right of zero and get larger as you go further right. Negative numbers are always to the left of zero and get smaller (or more negative) as you go further left.

Lesson 2**

Adding Numbers on a Number Line.

Today, we're going to learn how to place numbers on our number line. Get ready to be a number line expert!

Step 1: Draw Your Number Line

First, grab a piece of paper and a pencil. Draw a straight, horizontal line. This is your number line. It represents all the numbers that exist, from the smallest negative number to the largest positive number. That's a lot of numbers!

Step 2: Mark the Middle Point

Now, let's find the center point on your line and make a small vertical mark. This is where the number zero (0) lives. Above or below this mark, write a "0." Great job! You've just placed your first number on the number line.

Step 3: Adding Positive Numbers

To the right of the zero, make more small vertical marks. Each of these marks represents a positive number. Above or below each mark, write the number it represents. Start with 1 for the first mark to the right of 0, then 2 for the second mark, and so on. You've just placed positive numbers on the number line!

Step 4: Adding Negative Numbers

The left side of zero is where the negative numbers live. Just like you did with the positive numbers, make small vertical marks to the left of the zero. These marks represent negative numbers. Start with -1 for the first mark to the left of 0, then -2 for the second mark, and so on. Fantastic! Now you have negative numbers on your number line!

Step 5: Placing Other Numbers

What if you have to place a number that's not a whole number, like a fraction or a decimal? Easy-peasy! Let's say you need to place $\frac{1}{2}$ or 0.5 on your number line. This number lives halfway between 0 and 1. So, find the space between 0 and 1 and make a mark in the middle. Write " $\frac{1}{2}$ " or "0.5" above or below it. Awesome! You've just placed a fraction and a decimal on your number line!

Remember: The number line is all about order. Positive numbers are always to the right of zero and get larger as you go further right. Negative numbers are always to the left of zero and get smaller (or more negative) as you go further left.

Lesson 3

Subtracting Numbers on a Number Line.

Today, we'll learn how to subtract numbers using a number line. Let's dive in!

Step 1: Draw Your Number Line

Start by drawing a straight horizontal line on a piece of paper. This will be your number line. Mark the center of your line as zero (0). To the right of zero, mark and label some positive numbers (1, 2, 3, and so on). To the left of zero, mark and label some negative numbers (-1, -2, -3, and so on). Your number line is ready to use!

Step 2: Start with the First Number

Let's say our problem is $5 - 3$. Start with the first number, which is 5 in this case. Find the number 5 on your number line. Put your pencil on it or use a small object to mark your starting point.

Step 3: Move Along the Number Line

The second number tells you how many steps to take. But here's where subtraction gets interesting! Instead of moving to the right like we do with addition, we'll move to the left. So, for our problem, we need to take 3 steps to the left from our starting point at 5.

Step 4: Find the Answer

After taking 3 steps to the left from 5, where do you land? You got it, you landed on 2! So, $5 - 3$ equals 2. Congratulations, you've just used a number line to solve a subtraction problem!

Remember: When we subtract, we move to the left on the number line. The number of steps we take is determined by the second number in our problem.

Lesson 4

Identifying Natural Numbers.

Today we are going to learn about a very important concept in mathematics called “Natural Numbers.”

Understanding natural numbers is fundamental for all the math we do. They are numbers that we use every day without even realizing it, and they have been used for thousands of years for counting and ordering things.

Definition:

Natural numbers are the basic counting numbers that we learn when we first start counting things. They are the numbers that start from 1 and go on to 2, 3, 4, 5, and so on, up to infinity. We can write them as:

Natural Numbers = {1, 2, 3, 4, 5, 6, 7, 8, 9,}

Remember, natural numbers do not include zero, fractions, decimals, or negative numbers. They are always positive and whole.

Examples:

For example, if we were counting apples in a basket, we might say there are 1, 2, 3, or 10 apples, but we would not say there are 0, -2, or 1.5 apples.

Brief Activity:

Let's practice identifying natural numbers! Look at the following list of numbers and pick out the natural numbers:

{0, 1, 2, 3, -1, 4, 5.5, 6, -7, 8, 9.2}

Did you pick out 1, 2, 3, 4, 6, 8? Good job! Those are all natural numbers.

Lesson 5

Identifying Whole Numbers.

Today we're going to delve into an important concept in mathematics - "Whole Numbers."

Just like natural numbers, understanding whole numbers is fundamental to everything we do in math. They are the numbers we use constantly, often without even realizing it!

Definition:

Whole numbers are very similar to natural numbers, but with one key addition: the number 0. That's right! Whole numbers are all the natural numbers plus zero. They are the numbers that start from 0 and go on to 1, 2, 3, 4, 5, and so on, up to infinity. We can write them as:

Whole Numbers = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...}

Keep in mind, whole numbers do not include fractions, decimals, or negative numbers. They are always non-negative and whole.

Examples:

For example, if we were counting how many books are in a bag, we might say there are 0, 1, 2, 3, or 10 books, but we would not say there are -2, or 1.5 books.

Brief Activity:

Now, let's practice identifying whole numbers! Look at the following list of numbers and pick out the whole numbers:

{-1, 0, 1, 2.5, 3, -4, 5, 6, 7.7, 8, -9}

Did you pick out 0, 1, 3, 5, 6, 8? Excellent! Those are all whole numbers.

Lesson 6

Identifying Integers.

Today we are going to learn about a very important concept in mathematics called “Natural Numbers.”

Understanding natural numbers is fundamental for all the math we do. They are numbers that we use every day without even realizing it, and they have been used for thousands of years for counting and ordering things.

Definition:

Natural numbers are the basic counting numbers that we learn when we first start counting things. They are the numbers that start from 1 and go on to 2, 3, 4, 5, and so on, up to infinity. We can write them as:

Natural Numbers = {1, 2, 3, 4, 5, 6, 7, 8, 9,}

Remember, natural numbers do not include zero, fractions, decimals, or negative numbers. They are always positive and whole.

Examples:

For example, if we were counting apples in a basket, we might say there are 1, 2, 3, or 10 apples, but we would not say there are 0, -2, or 1.5 apples.

Brief Activity:

Let's practice identifying natural numbers! Look at the following list of numbers and pick out the natural numbers:

{0, 1, 2, 3, -1, 4, 5.5, 6, -7, 8, 9.2}

Did you pick out 1, 2, 3, 4, 6, 8? Good job! Those are all natural numbers.

Lesson 7

Identifying Rational Numbers.

Today we're going to explore another very important concept in mathematics - "Integers."

Integers may sound like a fancy term, but it's just another word for something we're already familiar with. Today, we'll learn what integers are, how we can identify them, and how they're used in math.

Definition:

So, what exactly are integers?

Integers are all the whole numbers AND their negatives. This means we include all the positive whole numbers, zero, and also all the negative whole numbers. We can write them as:

Integers = $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Notice how there's no end to how many integers there are. They go on forever in both directions!

Unlike natural and whole numbers, integers can be negative. But just like natural and whole numbers, they do not include fractions or decimals.

Examples:

For example, the temperature outside can be -5 degrees, 0 degrees, or 15 degrees. These are all examples of integers.

Brief Activity:

Let's practice identifying integers! Look at the following list of numbers and pick out the integers:

$\{-3, -2, -1.5, 0, 1, 2.5, 3, -4, 5.5, 6, -7\}$

Did you pick out -3, -2, 0, 1, 3, -4, 6, -7? Great job! Those are all integers.

Lesson 6

Identifying Rational Numbers.

Today we're going to venture further into the world of numbers and discover an important category of numbers called "Rational Numbers."

The term "Rational Numbers" might sound a bit complex, but don't worry, it's just a fancy name for numbers that can be expressed in a particular way. Let's dive in and unravel what rational numbers are all about.

Definition:

Rational numbers are any numbers that can be expressed as the quotient (or division) of two integers, with the denominator not being zero. That's a bit of a mouthful, so let's break it down.

If we have two integers, say 3 and 4, and we divide 3 by 4, we get 0.75. That's a rational number! It can be written as $\frac{3}{4}$, a fraction, which is just one integer over another. We can write them as:

Rational Numbers = $\frac{a}{b}$, where a and b are integers, and $b \neq 0$

Rational numbers include all the integers, fractions, mixed numbers, and finite decimal numbers or repeating decimals.

Examples:

Examples of rational numbers include $\frac{1}{2}$ (which is 0.5 when converted to a decimal), $-\frac{3}{4}$, 5 (which can be written as $\frac{5}{1}$), and 0.333... (which is $\frac{1}{3}$ as a fraction).

Brief Activity:

Now, let's practice identifying rational numbers! Look at the following list of numbers and pick out the rational numbers:

$\{\frac{1}{2}, 1.414, -3, \sqrt{2}, 4.25, 0, 0.777..., \pi\}$

Did you pick out $\frac{1}{2}$, -3, 4.25, 0, 0.777...? If you did, then you are absolutely correct! Those are all rational numbers.

Answers to Task Cards

Answers to Concept Cards

A-1

Concept presentation

A-2

Answers will vary

A-3

The given sequence appears to be an arithmetic sequence, where each number is 3 greater than the previous number. To find the next three numbers in this sequence, we add 3 to the last number provided, and continue doing so until we have the next three numbers.

$$30 \text{ (the last number in the sequence given)} + 3 = 33$$

$$33 \text{ (the next number in the sequence)} + 3 = 36$$

$$36 \text{ (the next number in the sequence)} + 3 = 39$$

So, the next three numbers in the sequence are 33, 36, and 39.

A-4

The given sequence appears to be an arithmetic sequence, where each number is 6 greater than the previous number. To find the next three numbers in this sequence, we add 6 to the last number provided, and continue doing so until we have the next three numbers.

$$33 \text{ (the last number in the sequence given)} + 6 = 39$$

$$39 \text{ (the next number in the sequence)} + 6 = 45$$

$$45 \text{ (the next number in the sequence)} + 6 = 51$$

So, the next three numbers in the sequence are 39, 45, and 51.

A-5

The given sequence appears to be an arithmetic sequence, where each number is 2 greater than the previous number. This can also be seen as increasing by 2.

$$10 \text{ (the last number in the sequence given)} + 2 = 12$$

$$12 \text{ (the next number in the sequence)} + 2 = 14$$

$$14 \text{ (the next number in the sequence)} + 2 = 16$$

So, the next three numbers in the sequence are 12, 14, and 16.

A-6

$$-1.5 \text{ or } -1\frac{1}{2}$$

$$1.5 \text{ or } 1\frac{1}{2}$$

$$2.5 \text{ or } 2\frac{1}{2}$$

A-7

Answers will vary. However, all reasonable answers where the tick marks are equally divided between the 0 and the final number should be accepted.

A-8

Answers will vary. However, all reasonable answers where the tick marks are equally divided between the 0 and the final number should be accepted.

A-9

Concept: Give lesson on Scale of a number line.

A-10

A: .1
B: .4
C: .55
D: .8
E: .9.5
F: .02
G: .04
H: .055
I: .07
J: .085

A-11

Concept Presentation

A-12

The fraction $1/8$ as a decimal is 0.125.
The decimal 0.25 as a fraction is $1/4$ or $25/100$.
The fraction $3/4$ as a decimal is 0.75.
The fraction $3/16$ as a decimal is approximately 0.1875.

A-13

$4 + 5 = 9$
 $2 + 6 = 8$
 $3 + 6 = 9$
 $4 + 3 = 7$
 $6 + 2 = 8$
 $1 + 8 = 9$
 $5 + 3 = 8$

A-14

$5 - 2 = 3$
 $6 - 2 = 4$
 $6 - 3 = 3$
 $3.5 - 2 = 1.5$
 $5.75 - 2.5 = 3.25$
 $8 - 1.25 = 6.75$
 $9 - 3 = 6$

A-15

$-4 + (-5) = -9$
 $-2 + (-6) = -8$
 $-3 + (-6) = -9$
 $-4 + (-3) = -7$
 $-6 + (-2) = -8$
 $-1 + (-8) = -9$
 $-5 + (-3) = -8$

A-16

$-5 - (-2) = -5 + 2 = -3$
 $6 - 2 = 4$
 $-6 - (-3) = -6 + 3 = -3$
 $-3.5 - (-2) = -3.5 + 2 = -1.5$
 $-5.75 - (-2.5) = -5.75 + 2.5 = -3.25$
 $-8 - (-1.25) = -8 + 1.25 = -6.75$
 $-9 - (-3) = -9 + 3 = -6$

A-17

To compare these negative fractions and arrange them from least to greatest, it's easier if you recognize that a fraction with a larger denominator (when the numerators are the same or proportional) is smaller. Also, when dealing with negative fractions, the fractions closer to zero are greater.

$-1/2$ is equivalent to $-6/12$
 $-3/8$ is equivalent to $-4.5/12$
 $-5/12$ is as it is.

So when arranged from least to greatest (or from most negative to least negative), we get:

$-1/2, -5/12, -3/8$

A different way to compare them is by converting them to a decimal. To do this divide the numerator by the denominator. This will give you the following:

$-1/2$ is equivalent to -0.5
 $-3/8$ is equivalent to -0.375

$-5/12$ is approximately -0.4167

Now you can arrange them from lowest to greatest. Remember that the number closest to 0 is greater.

A-18

Concept

A-19

Remember the opposite of a number is simply the number with the opposite sign. So, if a number is positive, its opposite is the same number, but negative, and vice versa. Here are the opposites of the numbers you provided:

The opposite of -5 is 5 .

The opposite of -2 is 2 .

The opposite of 6 is -6 .

The opposite of -3.75 is 3.75 .

The opposite of 5.25 is -5.25 .

The opposite of 8 is -8 .

The opposite of $-3/8$ is $3/8$.

A-20

The absolute value of a number is its distance from zero on the number line, so it's always nonnegative.

$$|12| = 12$$

$$|-15| = 15$$

$$|-1.2387| = 1.2387$$

$$|65| = 65$$

$$|-5.45| = 5.45$$

$$|24| = 24$$

Answers to Drill Cards

A-1

The given sequence appears to be an arithmetic sequence, where each number is 2 greater than the previous number. To find the next three numbers in this sequence, we add 2 to the last number provided, and continue doing so until we have the next three numbers.

$$\begin{aligned} 11 \text{ (the last number in the sequence given)} + 2 &= \\ 13 \\ 13 \text{ (the next number in the sequence)} + 2 &= 15 \\ 15 \text{ (the next number in the sequence)} + 2 &= 17 \end{aligned}$$

So, the next three numbers in the sequence are 13, 15, and 17.

A-2

This sequence doesn't follow a straightforward arithmetic or geometric pattern (where you add or multiply by the same number each time).

However, it appears that the sequence alternates between adding 2 and adding 1:

$$\begin{aligned} 2 + 2 &= 4, \\ 4 + 1 &= 5, \\ 5 + 2 &= 7. \end{aligned}$$

If this pattern continues, the next three numbers in the sequence would be calculated as:

$$\begin{aligned} 7 + 1 &= 8, \\ 8 + 2 &= 10, \\ 10 + 1 &= 11. \end{aligned}$$

So, the next three numbers in the sequence are 8, 10, and 11.

A-3

Answers will vary

A-4

5:

It is a natural number because it is a positive whole number greater than zero.

0:

It is a whole number because it is non-negative and does not have any fractional or decimal part. However, it is not considered a natural number since natural numbers are positive.

-2:

It is an integer because it is a whole number that can be negative.

7:

It is a natural number because it is a positive whole number greater than zero.

-10:

It is an integer because it is a whole number that can be negative.

13:

It is a natural number because it is a positive whole number greater than zero.

A-5

3:

It is a natural number because it is a positive whole number greater than zero.

It is a whole number because it does not have any fractional or decimal part.

It is an integer because it can be expressed

without a fractional or decimal part.
It is a rational number because it can be expressed as a fraction ($\frac{3}{1}$).

0:

It is a whole number because it does not have any fractional or decimal part.
It is an integer because it can be expressed without a fractional or decimal part.
It is a rational number because it can be expressed as a fraction ($\frac{0}{1}$).

-5:

It is an integer because it can be expressed without a fractional or decimal part.
It is a rational number because it can be expressed as a fraction ($-\frac{5}{1}$).

$\frac{2}{3}$:

It is a rational number because it can be expressed as a fraction.

-7.5:

It is a rational number because it can be expressed as a decimal fraction ($-7.5 = -\frac{15}{2}$).

10:

It is a natural number because it is a positive whole number greater than zero.
It is a whole number because it does not have any fractional or decimal part.
It is an integer because it can be expressed without a fractional or decimal part.
It is a rational number because it can be expressed as a fraction ($\frac{10}{1}$).

A-6

13:

It is a natural number because it is a positive whole number greater than zero.
It is a whole number because it does not have any fractional or decimal part.
It is an integer because it can be expressed without a fractional or decimal part.

It is a rational number because it can be expressed as a fraction ($\frac{13}{1}$).

-12:

It is an integer because it can be expressed without a fractional or decimal part.
It is a rational number because it can be expressed as a fraction ($-\frac{12}{1}$).

-0.876:

It is a rational number because it can be expressed as a decimal fraction.

$\frac{2}{16}$:

It is a rational number because it can be expressed as a fraction.

4.58:

It is a rational number because it can be expressed as a decimal fraction.

19:

It is a natural number because it is a positive whole number greater than zero.
It is a whole number because it does not have any fractional or decimal part.
It is an integer because it can be expressed without a fractional or decimal part.
It is a rational number because it can be expressed as a fraction ($\frac{19}{1}$).

A-7

A.

To determine the values of the tick marks, we need to divide the total range (36) by the number of segments. Given there are 9 tick marks, this will create 8 segments (since one less segment than tick marks).

36 divided by 8 equals 4.5.

This means each tick mark represents a value of 4.5 units more than the previous tick mark. Let's identify the tick marks:

The first tick mark is 0.
The second tick mark is 4.5.
The third tick mark is 9.0.
The fourth tick mark is 13.5.
The fifth tick mark is 18.0.
The sixth tick mark is 22.5.
The seventh tick mark is 27.0.
The eighth tick mark is 31.5.
The ninth tick mark is 36.0.

B.
An open number line that begins at 0 and ends at 18, with 3 tick marks, is divided into 4 equal sections (think of the tick marks as dividing the line into segments).

To find out the measurement or value represented by each section, you would divide the total range of the number line (from 0 to 18) by the number of sections:

$$18 \div 4 = 4.5$$

So, each section or distance between each tick mark on the number line represents a value of 4.5.

C.

If an open number line starts at 0 and ends at 76 with a total of 15 tick marks, we first need to understand that these tick marks divide the line into 16 equal sections (since the tick marks include the starting and ending points).

To find the value represented by each section, we should divide the total range of numbers (from 0 to 76) by the number of sections (16):

$$76 \div 16 = 4.75$$

So, each section on your number line represents a value of 4.75. Now, we can assign numbers to each of the tick marks. Remember, we start at 0:

1st tick mark: 0 (start point)

2nd tick mark: 4.75
3rd tick mark: 9.5
4th tick mark: 14.25
5th tick mark: 19
6th tick mark: 23.75
7th tick mark: 28.5
8th tick mark: 33.25
9th tick mark: 38
10th tick mark: 42.75
11th tick mark: 47.5
12th tick mark: 52.25
13th tick mark: 57
14th tick mark: 61.75
15th tick mark: 66.5
16th tick mark: 71.25
17th tick mark: 76 (end point)

A-8

Fraction: $\frac{5}{2}$, Decimal Equivalent: 2.5
Fraction: $\frac{3}{2}$, Decimal Equivalent: 1.5
Fraction: $\frac{7}{3}$, Decimal Equivalent: Approximately 2.33 (rounded to 2 decimal places)
Fraction: $\frac{19}{2}$, Decimal Equivalent: 9.5
Fraction: $\frac{8}{3}$, Decimal Equivalent: Approximately 2.67 (rounded to 2 decimal places)
Fraction: $\frac{15}{2}$, Decimal Equivalent: 7.5
Fraction: $\frac{11}{2}$, Decimal Equivalent: 5.5
Fraction: $\frac{17}{3}$, Decimal Equivalent: Approximately 5.67 (rounded to 2 decimal places)
Fraction: $\frac{9}{2}$, Decimal Equivalent: 4.5
Fraction: $\frac{13}{3}$, Decimal Equivalent: Approximately 4.33 (rounded to 2 decimal places)

A-9

Fraction: $\frac{4}{1}$, Decimal Equivalent: 4.0
Fraction: $\frac{7}{1}$, Decimal Equivalent: 7.0
Fraction: $\frac{13}{2}$, Decimal Equivalent: 6.5
Fraction: $\frac{18}{1}$, Decimal Equivalent: 18.0
Fraction: $\frac{11}{2}$, Decimal Equivalent: 5.5
Fraction: $\frac{15}{1}$, Decimal Equivalent: 15.0
Fraction: $\frac{19}{1}$, Decimal Equivalent: 19.0
Fraction: $\frac{17}{2}$, Decimal Equivalent: 8.5

Fraction: $9/1$, Decimal Equivalent: 9.0
Fraction: $14/2$, Decimal Equivalent: 7.0

Arranged in order would be:

4.0
5.5
6.5
7.0
7.0
8.5
9.0
15.0
18.0
19.0

A-10

Fraction: $1/4$, Decimal Equivalent: 0.25
Fraction: $3/4$, Decimal Equivalent: 0.75
Fraction: $2/5$, Decimal Equivalent: 0.40
Fraction: $3/5$, Decimal Equivalent: 0.60
Fraction: $7/10$, Decimal Equivalent: 0.70
Fraction: $13/20$, Decimal Equivalent: 0.65
Fraction: $17/25$, Decimal Equivalent: 0.68
Fraction: $29/50$, Decimal Equivalent: 0.58
Fraction: $33/50$, Decimal Equivalent: 0.66
Fraction: $9/16$, Decimal Equivalent: 0.56

A-11

Decimal: 0.2, Fractions: $1/5$, $2/10$
Decimal: 0.5, Fractions: $1/2$, $5/10$
Decimal: 0.75, Fractions: $3/4$, $75/100$
Decimal: 0.1, Fractions: $1/10$, $10/100$
Decimal: 0.8, Fractions: $4/5$, $80/100$
Decimal: 0.6, Fractions: $3/5$, $60/100$
Decimal: 0.4, Fractions: $2/5$, $40/100$
Decimal: 0.9, Fractions: $9/10$, $90/100$
Decimal: 0.3, Fractions: $3/10$, $30/100$
Decimal: 0.7, Fractions: $7/10$, $70/100$
Note that each pair of fractions are equivalent, meaning they represent the same decimal value.

A-12

Decimal: 26.35, Fractions: $2635/100$, $527/20$
Decimal: 32.70, Fractions: $327/10$, $3270/100$
Decimal: 40.25, Fractions: $401/10$, $802/20$
Decimal: 45.85, Fractions: $4585/100$, $917/20$
Decimal: 52.15, Fractions: $5215/100$, $1043/20$
Decimal: 57.75, Fractions: $5775/100$, $1155/20$
Decimal: 64.50, Fractions: $645/10$, $1290/20$
Decimal: 68.25, Fractions: $6825/100$, $1365/20$
Decimal: 70.60, Fractions: $706/10$, $1412/20$
Decimal: 72.90, Fractions: $729/10$, $1458/20$
In each case, the two fractions are equivalent to the decimal. The fraction is simplified by dividing both the numerator and the denominator by their greatest common factor.

A-13

$10 + 5 = -5$
 $7 + 8 = 15$
 $-6 + 10 = 4$
 $-15 + 20 = 5$
 $10 + 10 = 20$
 $-8 + -7 = -15$
 $12 + 6 = 18$
 $-10 + -5 = -15$
 $15 + 3 = 18$
 $-20 + 18 = -2$

A-14

$-20 + 10 = -10$
 $-5 + 8 = 3$
 $-25 + 20 = -5$
 $-15 + 30 = 15$
 $10 + 5 = 15$
 $-10 + -10 = -20$
 $12 + 3 = 15$
 $-20 + -5 = -25$
 $15 + -5 = 10$
 $-30 + 10 = -20$

A-15

$$\begin{aligned}
 -20 + 10 &= -10 \\
 -5 + 8 &= 3 \\
 -25 + 20 &= -5 \\
 -15 + 30 &= 15 \\
 10 + 5 &= 15 \\
 -10 + -10 &= -20 \\
 12 + 3 &= 15 \\
 -20 + -5 &= -25 \\
 15 + -5 &= 10 \\
 -30 + 10 &= -20
 \end{aligned}$$

A-16

$$\begin{aligned}
 30 - 10 &= 20 \\
 -15 - 5 &= -20 \\
 50 - 25 &= 25 \\
 -20 - 10 &= -30 \\
 45 - 40 &= 5 \\
 -25 - 10 &= -35 \\
 50 - 10 &= 40 \\
 -30 - -10 &= -20 \\
 50 - -20 &= 70 \\
 -40 - -30 &= -10
 \end{aligned}$$

A-17

$x > -5$: All numbers greater than -5
 $x < 10$: All numbers less than 10
 $x \geq 0$: All numbers greater than or equal to 0
 $x \leq -10$: All numbers less than or equal to -10
 $x > 15$: All numbers greater than 15
 $x < -15$: All numbers less than -15
 $x \geq -20$: All numbers greater than or equal to -20
 $x \leq 20$: All numbers less than or equal to 20
 $x > 0$: All numbers greater than 0
 $x < 0$: All numbers less than 0

A-18

$x > -4.5$: All numbers greater than -4.5
 $x < 12.5$: All numbers less than 12.5
 $x \geq 0.75$: All numbers greater than or equal to 0.75
 $x \leq -9.5$: All numbers less than or equal to -9.5
 $x > 1/2$: All numbers greater than $1/2$
 $x < -3/4$: All numbers less than $-3/4$
 $x \geq -15$: All numbers greater than or equal to -15
 $x \leq 2.5$: All numbers less than or equal to 2.5
 $x > -17.5$: All numbers greater than -17.5

$x < 7/8$: All numbers less than $7/8$

A-19

$$\begin{aligned}
 |5| + |-3| &= 8 \\
 |-10| - |5| &= 5 \\
 |4| * |-2| &= 8 \\
 |-12| / |3| &= 4 \\
 |-7| + |-8| &= 15 \\
 |9| - |-5| &= 4 \\
 |2| * |-6| &= 12 \\
 |-16| / |-4| &= 4 \\
 |-5| + |3| &= 8 \\
 |-20| / |5| &= 4
 \end{aligned}$$

A-20

$$\begin{aligned}
 |2/3| + |-1/2| &= 1.17 \text{ (approx)} \\
 |-3/4| - |1/2| &= 0.25 \\
 |1/3| * |-2/3| &= 0.22 \text{ (approx)} \\
 |-2/5| / |4/5| &= 0.5 \\
 |-7/9| + |-5/6| &= 1.61 \text{ (approx)} \\
 |5/7| - |-3/4| &= 0.03 \text{ (approx)} \\
 |2/3| * |-3/5| &= 0.4 \\
 |-5/6| / |-3/4| &= 1.13 \text{ (approx)} \\
 |-3/2| + |4/3| &= 2.17 \text{ (approx)} \\
 |-8/9| / |2/3| &= 1.33 \text{ (approx)}
 \end{aligned}$$



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