

# Upper Elementary



## Math Task Cards

Teacher's Notes and Manual



# Upper Elementary Advanced Math

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# Lattice Multiplication

## Age

10 years

## Aim

Direct: to explore different forms of multiplication

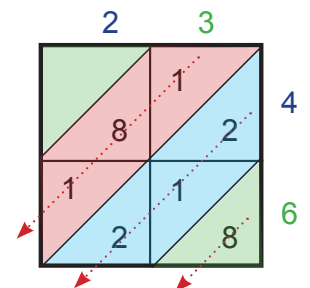
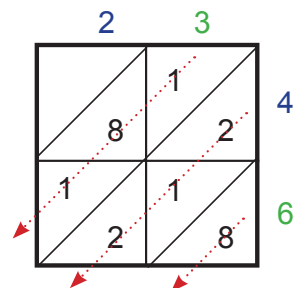
Indirect: to refine mental math and place value skills

## Materials

Paper and pencil, task card

## Presentation

1. Invite the child to the lesson.
2. "This is a different way to multiply. You have learned what is termed *long multiplication*, where you write each partial product and then add them together to calculate your total product."
3. "This is simpler form of multiplication. In the late 13th century, Arabic traders brought this method to Italy, and it spread throughout Europe."
4. "A lattice is a structure of crossed wooden or metal strips. It is usually arranged to form a diagonal pattern of open spaces between the strips."
5. Draw the lattice grid on the paper for  $23 \times 46$ .
6. "Multiply by your units first. Six times 3 equals 18. Place the 8 in the lower grid and the one in the upper grid. Six times 2 is 12. Place the 2 in the lower grid and the one in the upper grid. Four times 2 is 8. Place the 8 in the lower grid.."
7. "Now, let's multiply by our tens. Four times three is twelve. Place the two in the lower grid and the one in the upper grid. Four times two is eight. Place the eight in the lower grid."
8. "We need to calculate the total product. Look at the digits along the diagonals."
9. "Look at the unit's place. There is an 8 in the bottom grid. This represents the digit for the unit's place."
10. "In the ten's place we have  $2 + 1 + 2$ . This equals 5. This represents the digit in the ten's place."
11. "Look at the hundred's place. We have  $1 + 8 + 1$ . This equals 10, which means we have a 0 in the hundred's place and a 1 in the thousand's place."
12. "The total product is 1,058."



## Follow Up

1. Multiplication of decimals

# Napier's Bones

## Age

9-11 years

## Aim

Direct: to learn an historic form of calculation  
Indirect: to work on mental math and place value

## Materials

A set of Napier's Bones, a multiplication problem

## Presentation 1

Index	1	2	3	4	5	6	7	8	9	0
1	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	0 0
2	0 2	0 4	0 6	0 8	1 0	1 2	1 4	1 6	1 8	0 0
3	0 3	0 6	0 9	1 2	1 5	1 8	2 1	2 4	2 7	0 0
4	0 4	0 8	1 2	1 6	2 0	2 4	2 8	3 2	3 6	0 0
5	0 5	1 0	1 5	2 0	2 5	3 0	3 5	4 0	4 5	0 0
6	0 6	1 2	1 8	2 4	3 0	3 6	4 2	4 8	5 4	0 0
7	0 7	1 4	2 1	2 8	3 5	4 2	4 9	5 6	6 3	0 0
8	0 8	1 6	2 4	3 2	4 0	4 8	5 6	6 4	7 2	0 0
9	0 9	1 8	2 7	3 6	4 5	5 4	6 3	7 2	8 1	0 0

1. Invite the child to the lesson.
2. "John Napier was a Scottish mathematician who lived from 1550 to 1617. He was educated at home until age 13, at which time he entered the University of St. Andrews."
3. "Napier is best known for his work on logarithms. He is quoted as saying, '*There is nothing so troublesome to mathematical practice than multiplications, divisions, square and cubical extractions of great numbers.... I began therefore to consider how I might remove those hindrances.*' These 'bones' allowed him to do calculations quickly. Long ago, the rods were made of bone, which is why they have this name."
4. "Let's use the bones to multiply 846 by 6."
5. Lay out the following rods to build your multiplicand with the index rod to the left.

Index	8	4	6
1	0 8	0 4	0 6
2	1 6	0 8	1 2
3	2 4	1 2	1 8
4	3 2	1 6	2 4
5	4 0	2 0	3 0
6	4 8	2 4	3 6
7	5 6	2 8	4 2
8	6 4	3 2	4 8
9	7 2	3 6	5 4

6. "Your multiplier is 6. Look across the 6 line at the diagonals and add the numbers **within** the diagonals."
7. "Your product will be 5,076."

## Follow Up

1. Task cards
2. Work with multi-digit multipliers.

## Presentation 2

1. Invite the child to the lesson.
2. "You can use Napier's Bones for multi-digit multipliers as well. Let's multiply 691 by 13."
3. Lay out the following rods to build your multiplicand with the index rod to the left.

Index	6	9	1
1	0 6	0 9	0 1
2	1 2	1 8	0 2
3	1 8	2 7	0 3
4	2 4	3 6	0 4
5	3 0	4 5	0 5
6	3 6	5 4	0 6
7	4 2	6 3	0 7
8	4 8	7 2	0 8
9	5 4	8 1	0 9

4. "Your multiplier is 13. Look across the 3 line at the diagonals and add the numbers **within** the diagonals for your first partial product."
5. "Look across the 1 line at the diagonals and add the numbers **within** the diagonals for your second partial product. Remember to put a zero in the unit's place when you record your second partial product."
6. "Add your 2 partial products and your total product will be 8,983 ."

## Follow Up

1. Task cards
2. Work with division

## Presentation 3

1. Invite the child to the lesson.
2. "The bones can also be used to divide. Let's take  $57,268 \div 278$ ."
3. Lay out the 2, 7, and 8 rods to build your divisor with the index rod to the left.
4. "Use the digits on the diagonals to produce the products of:

$$\begin{aligned} &278 \times 1 \\ &278 \times 2 \\ &278 \times 3 \\ &278 \times 4 \\ &278 \times 5, \text{ etc.} \end{aligned}$$

5. "Let's use these sums to subtract from our dividend."
6. "Since we have a 3-digit divisor, we need to look for a number that will go into the first 3 digits."
7. "The closest is 556, which is the product of 278 times 2. Place a 2 in the hundred's place in the quotient. When you subtract 556 from 572 you are left with 16."
8. "Bring down the next digit, which gives us 166. Since there is not a sum that will go into 166, we have a zero in the ten's place of our quotient."
9. "Subtract zero from 166, which leaves 166. Bring down the next digit, making 1,668."
10. "The sum that is a product of 278 and 6 is 1,668. Place a 6 in the unit's place of the quotient and subtract 1,668, which leaves zero."
11. "The quotient is 206."

Index	2	7	8	
1	0 2	0 7	0 8	278
2	0 4	1 4	1 6	556
3	0 6	2 1	2 4	834
4	0 8	2 8	3 2	1112
5	1 0	3 5	4 0	1390
6	1 2	4 2	4 8	1668
7	1 4	4 9	5 6	1946
8	1 6	5 6	6 4	2224
9	1 8	6 3	7 2	2502

$$\begin{array}{r} 278 \overline{) 57,268} \\ \underline{556} \phantom{0} \\ 16 \phantom{6} \\ \phantom{16} \downarrow \\ \phantom{16} 6 \\ \phantom{16} \phantom{6} \downarrow \\ \phantom{16} \phantom{6} 0 \\ \phantom{16} \phantom{6} \phantom{0} \downarrow \\ \phantom{16} \phantom{6} \phantom{0} 6 \\ \phantom{16} \phantom{6} \phantom{0} \phantom{6} \downarrow \\ \phantom{16} \phantom{6} \phantom{0} \phantom{6} 8 \\ \phantom{16} \phantom{6} \phantom{0} \phantom{6} \phantom{8} \downarrow \\ \phantom{16} \phantom{6} \phantom{0} \phantom{6} \phantom{8} 0 \end{array}$$



# Negative Numbers

## Age

10-11 years

## Aim

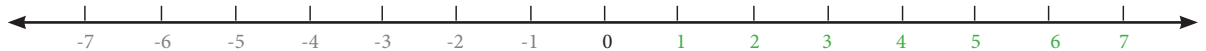
Direct: to recognize and place negative numbers in relation to positive numbers

Indirect: to understand the 4 operations with negative numbers

## Materials

number line with positive and negative numbers

## Presentation



1. Invite the child to the lesson.
2. "When people first started using numbers, they counted using what we call **real** numbers. These are the numbers that we see on our number line, starting at one."
3. Indicate this on the number line.
4. "Later, zero was added to the number system to be used as a place holder in different operations. This added the set of **whole** numbers to our classification of numbers."
5. "As traders, scribes, and merchants began to record their transactions, they discovered they needed to record amounts owed them. This brought about a new set of numbers, which we call negative numbers."
6. "Can you think of instances where negative numbers are used?" (temperatures below 0, money owed, elevation below sea level)
7. "When we place these values on a number line, they are on the opposite side of the positive numbers; just like the positive numbers, they go on infinitely. This gives us a new classification of numbers termed **integers**."
8. Indicate the negative numbers on the number line.
9. "Remember, zero is neither positive nor negative."
10. "We can measure the space between any integer and zero. This is called the **absolute value** of a number."
11. "We record the absolute value of a number in this way,  $|8|$ , and say, 'The absolute value is 8'."

## Follow Up

1. Task card C8 in Upper Elementary Advanced Math
2. Task cards 9 -12 in 9-12 Number Line Concepts



# Alternative Forms of Multiplication

## Age

9-10 years

## Aim

Direct: to learn alternative forms of multiplication

Indirect: to refine mental math and place value skills

## Cross Multiplication

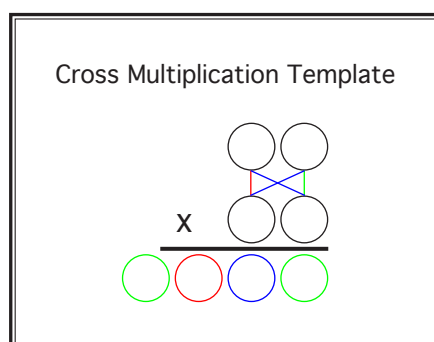
## Materials

A set of strips that indicate the products by place value of different multiplication problems, cross multiplication template, whole number checkerboard, multiplication problem

## Presentation

1. Invite the child to the lesson.
2. "Remember when you were first learning the checkerboard, we called out the place value of each digit. We said, units times units is units, units times tens is tens."
3. Point to the place value on the checkerboard as this is said.
4. "We have labels to help us remember our different place values. What are all the ways to make a unit as the product? Yes, there is just one, a unit times a unit."
5. Place the label on the rug.
6. "What are all the ways to make a ten as a product?" Lay out each strip as the child calls out the place value.
7. "Do you notice a pattern? How many ways do you think there are to make a hundred as a product? Yes, there are 3. Name all the ways to make a hundred as a product."
8. Continue exploring the different ways to make thousands, ten thousands, hundred thousands, and millions. Lay out each place value strip as the child names the different ways.
9. "We can use this information to do multiplication in a different way."
10. Pull out the cross multiplication template for multiplying a two digit multiplicand by a two digit multiplier.

units x units = units
units x tens = tens
tens x units = tens
units x hundreds = hundreds
tens x tens = hundreds
hundreds x units = hundreds



11. "You can see where we get the name cross multiplication. If you look at the different ways to make the products they make crosses."
12. "Let's do the problem  $32 \times 26$ . To get the unit, multiply 2 by 6. Two times 6 is 12. Write down the 2 and remember the one."
13. "To fill the ten's place, multiply 6 times 3. This equals 18. Multiply the 2 by 2, which equals 4. Add  $18 + 4 + 1$ , which equals 23. Write the 3 in the ten's place and remember the 2."
14. "To make the hundreds multiply 2 times 3. This makes 6. Add  $6 + 2 = 8$ ."
15. "The total product is 832."

# Bones the Card Game

*Money be the muscle of our right strong arm, but  
figures be the BONE!*

*Anon*

## I. How to Start

- You can play Bones with 2 to 4 players. It may also be played as a game of solitaire.
- Decide who will go first and then all play moves in a clockwise progression.
- Place the shuffled cards in 3 piles, side-by-side, **face up** between the players in the following order:
  - Blue bordered numeral cards on the left
  - Green bordered common symbol cards in the middle
  - Red bordered special symbol cards on the right
- Each player draws his cards from one stack (any stack he wishes) in the following way, and then the next player draws his cards.
  - Draw 5 cards at a time from the blue cards,
  - 4 at a time from the green, and
  - 3 at a time from the red

## II. How to Play

- The object of the game is to build a true mathematical equation using as many cards as possible.
- Place your cards in front of you and rearrange the cards making a balanced equation.
- Once you think you have made the best equations your cards allow, you can lay leftover cards on other players' equations as long as the extended equations are still true.
- Both people will get the extra points that result.

## III. How to Score

Balancing Symbol	Multiplying Value	Example	Points for Equations
$\neq$ Not equal	1x	$1 \neq 2$	$1 \times 2 \text{ cards} = 2$
$>$ Greater than	2x	$2 > 1$	$2 \times 2 \text{ cards} = 4$
$<$ Less than	2x	$1 < 2$	$2 \times 2 \text{ cards} = 4$
$\cong$ Approximately Equal	2x	$3 \cong \Pi$	$3 \times 2 \text{ cards} = 6$
$=$ Equal	4x	$1 = 1$	$4 \times 2 \text{ cards} = 8$

### III. How to Score (continued)

Some equations are basically more interesting than others. These will be worth twice the points as multiplying cards by their balancing value.

Equation	Point value	Explanation
$1 \div \infty = 0$	32	$\infty$ stands for infinity which is incredibly larger than any numeral
$1 \times \infty = 2 \times \infty$	48	Ancient Egyptians actually used this pi approximation in buildings
$\Pi = 3.14$	40	
$i = \sqrt{-1}$	32	imaginary numbers are represented by $i$
$! 4 = 1 \times 2 \times 3 \times 4$	72	! stands for a factorial calculation, a simple math procedure.
$! 3 = 1 \times 2 \times 3$	56	$^$ stands for raising to a power. For example $3^2$ is $3^2$ meaning $3 \times 3 = 9$
$\sqrt{1} = -1 \wedge 2 = 1$	56	

See if there are other “elegant” equations that you can make using the special symbols.

### III. Final Notes

- Cards may be flipped to show a different value. 6's can become 9's,  $\infty$  can be an 8.
- After 3 rounds, the player with the most points wins. Refer to the point ranges to see your pirate rank.
- Remember to shuffle the cards between rounds.

Points	Pirate Rank
100 points and up	Captain
67 to 99 points	First Mate
31 to 66 points	Able Bodied Seaman
up to 30 points	Swabby

# Algebraic Equations

## Age

10 - 11 years

## Aim

Direct: to learn the vocabulary associated with algebraic equations

Indirect: to solve algebraic equations

## Materials

white board, marker

## Presentation

1. Invite the child to the lesson.
2. "When a mathematical statement has an unknown value, use a letter to represent the unknown. This idea came from French mathematician Francois Viète."
3. "When I have an unknown value of something I can represent it with the letter **x**. This is termed a *variable*."
4. Record this on the white board
5. "If I have a certain number of unknown values, say 3 for example, I can write it as **3x**." The number in front of the variable is termed a *coefficient*."
6. Record this on the white board.
7. "A mathematical statement that has a variable and an operation sign is called an expression.  $3x + 4$  is an expression."
8. Record this on the white board.
9. "Finally, a mathematical expression that has an equal sign is called an equation.  $3x + 4 = 10$  is an equation."
10. Record this on the white board.



11. "In order to solve an equation it is useful to think of it as if it were on a balance scale."
12. Draw a balance scale on the white board.
13. "Think of the 2 parts of the equation that are separated by the equal sign as sitting in the scale's 2 pans. The equal sign is the fulcrum of the balance scale."
14. Record the 2 parts of the equation in each of the pans on the drawing and the = on the fulcrum.
15. "There are 3 rules to help solve an equation. The first rule is all of your variables should be on one side of the equal sign and all your numbers on the other side."
16. "The second rule is that what ever you do to one side of the equation, you must do to the other

side of the equation to keep the scale in balance.”

17. “The third rule is that the answer is the value of one variable. Let’s take a look at the equation that we already have.  $3x + 4 = 10$ .”
18. “I need to have all my variables on one side of the equal sign and all my numbers on the other side. How can I do that?”
19. “If the 4 is taken away from the left side of the equation, that will isolate the variables. The problem is that now my scale is off balance. To restore the balance, I need to take 4 away from the other side of the equation.”

$$\begin{array}{r} 3x + 4 = 10 \\ - 4 = 10 - 4 \\ \hline 3x = 6 \end{array}$$

20. “We want the value of one  $x$ . In order to get the value of one  $x$  we need to divide the  $3x$  by 3. However, this will throw the scale off balance again. To keep everything in balance we have to divide the other side of the equation by 3 as well.”

$$\begin{array}{r} \frac{3x}{3} = \frac{6}{3} \\ x = 2 \end{array}$$

21. “There is one last step. To check our work we will substitute the value of  $x$  into the original equation and solve. If the solution is a true statement, the value of  $x$  is correct.”

$$\begin{array}{r} 3(2) + 4 = 10 \\ 6 + 4 = 10 \\ 10 = 10 \quad \checkmark \end{array}$$

## Follow Up

1. More practice with solving for  $x$
2. Task cards

# Operations With Integers

## Age

10 -11 years

## Aim

Direct: to understand operations using negative numbers

Indirect: to prepare for pre-algebra concepts

## Materials

set of operational signs, set of tiles that are green on one side and gray on the other, picture set of green skittles, picture set of gray skittles, slips of paper, negative number line

## Presentation 1 - Addition and Subtraction (D 4-10)

1. Invite the child to the lesson.
2. "Today, we are going to use a new piece of material. You will notice that these tiles are two sided. The green side represents a positive quantity and the gray side represents a negative quantity."
3. Place 3 green sided tiles and five green sided tiles with a plus sign between them.



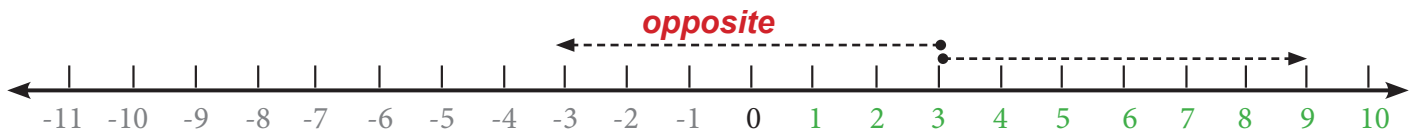
4. "This would be read as three plus five. Record this expression in your notebook and solve."
5. Provide the next example using a subtraction sign and ask the child to record and solve. Seven minus five. Record  $7 - 5 = 2$
6. "What would happen if we took  $-4 + -3$ ?"
7. Build the equation with the tiles.
8. "When we have a plus sign it means 'put them together'. Grouping the  $-4 + -3$  will give  $-7$ ."
9. "Record this equation in your math notebook."
10. "Let's make this a little more interesting. How would you read the following expression?"



11. "This is negative two plus positive 4. It is written as  $-2 + 4$  or  $-2 + (+4)$ . What do you think is the solution? If we take one negative tile and pair it with a positive tile, this is negative one plus positive one. They will cancel each other out and make a **zero pair**."
12. "The solution to the expression is two. Record this equation in your math notebook."
13. Build the last equation and ask the child to solve.  $3 + -6 =$ . Using the concept of zero pairs the child will arrive at the solution of negative three.



14. "On a number line it will look like this. I have three but I add six negatives. When I add I go to the right on a number line. I can also think of the negative sign as the opposite of addition."



## Presentation 2 - Adding Zero Pairs to Subtract a Positive (F 1-3)

1. Invite the child to the lesson.
2. "Today, we are going to use our strategy of adding zero pairs to help us with a subtraction equation."
3. Build the following expression.  $-3 - 8$ .



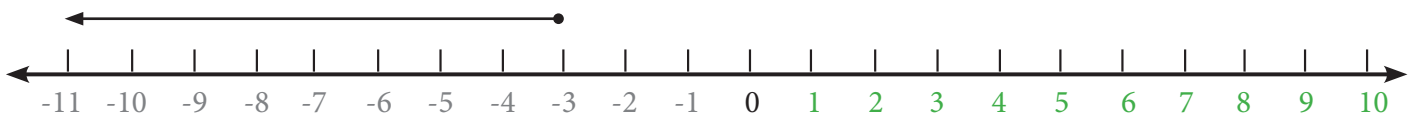
4. "We don't actually have eight positive to take away. However, if we add eight zero pairs it will not change anything."



5. "Now, we can take away the 8 positives. When we do that we are left with 11 negatives. Record this in your math notebook. "



6. Bring out the number line. "We can also show this on the number line. When ever we add on the number line we move to the right a certain number of spaces. Whenever we subtract on the number line we move to the left a certain number of spaces."
7. "If we start at -3 on the number line and we need to subtract 8, that means we will move to the left eight spaces."
8. Model this on the number line.
9. "When we count eight spaces to the left, we end up at -11."



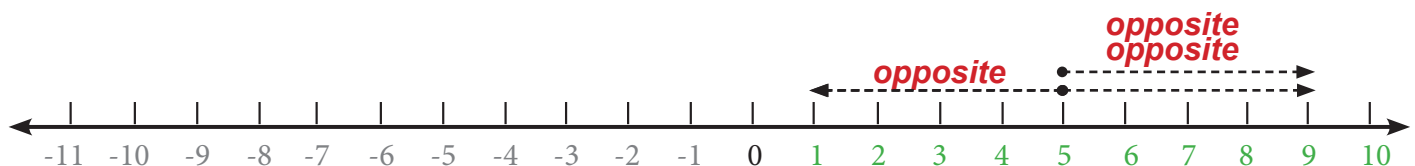


### Presentation 3 - Subtracting a Negative Number (F 1-5)

1. Invite the child to the lesson.
2. "We can also use the same strategy of adding zero pairs when we have to subtract a negative number. Let's look at the expression of  $5 - (-4)$ ."
3. Build the positive five. "We, have positive five, but we do not have any negatives to take away. However, if we add zero pairs, it will not change the value of what we have."

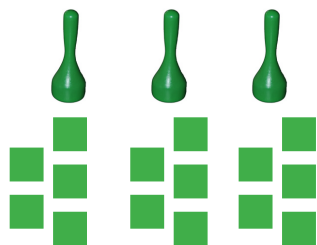


4. "This leaves us with +9. Record this equation in your math notebook."
5. "We can do this same problem with a number line. The only difference is that we are subtracting a negative number."
6. "Before, when we were subtracting, we moved to the left. We went in the **opposite** direction of addition. We can think of the negative sign as an **opposite** as well."
7. "We start at positive 5. We need to subtract so that means that we will go to the left in the opposite of addition. However, we have a negative, which is another opposite"
8. "So what we are doing is we are going **opposite, opposite**. This will actually make us go to the right."
9. Model this on the number line.
10. "We see the same thing in language when we have double negatives. If I say "Sit!" I am telling you to sit (positive)."
11. "But if I say, "Do not sit!" I am saying I want you to stand, not sit (negative)."
12. Now if I say, "Do NOT not sit!" I am saying I don't want you to stand, so I am back to saying sit (positive). So, two negatives make a positive."
13. As the child continues to work with negative numbers they will begin to formulate rules. Both visuals are used to help child to abstract these rules as some operations are easier with the number line, and some operations are easier with the tiles.



## Presentation 4 - Division with Negative Integers (G 1-4)

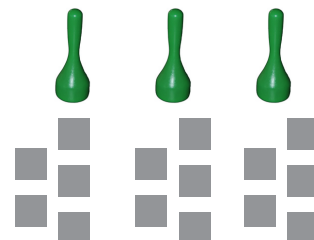
1. Invite the child to the lesson.
2. "Remember, when we divided we used skittles to represent our divisor? The quotient was always what one whole unit received."



3. Model the following problem with the picture skittles and the tiles, fifteen divided by three.  $15 \div 3$

4. "Therefore,  $15 \div 3 = 5$ . Record this in your math notebook."

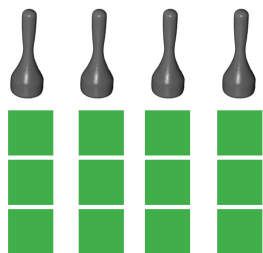
5. "This time, we are going to use our positive skittles to make the following problem. Negative fifteen divided by three.  $-15 \div 3$ ."



6. "The quotient in division is what one whole unit receives. Each unit skittle received -5. Record this in your math notebook."

7. "This time we will do something different. Instead of using a positive skittle, we will use a negative skittle."

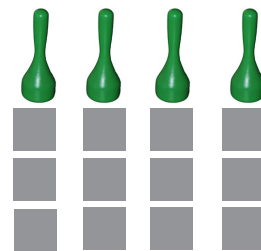
8. Take out the picture of the negative skittle and use it to model the following problem. Twelve divided by negative four.  $12 \div -4 =$



9. "We have a problem. The quotient is what one whole **positive** unit receives. This is not a positive unit. Remember, before when we were working with the number line? We said we could think of the negative sign as the **opposite**. Let's use that strategy here."

10. "We need to take the opposite. So, if the quotient of one negative unit is positive three, then the quotient for one positive skittle would be negative three."

11. Replace the negative skittles with the picture of the positive skittles and replace the green tiles with the gray tiles. Have the child write this statement in their math notebook.  $12 \div -4 = -3$ ."



## Presentation 5 - Multiplication with Negative Integers (G 5-10)

1. Invite the child to the lesson.
2. "When we are multiplying sets of negative numbers it is just like our lesson on multiplying sets of positive numbers. In the following problem,  $-3 \times 4$ , we need to think of it as a set of -3 taken four times."
3. Build that set with the tiles.



4. "So, a set of negative three taken four times equals negative twelve. Record this equation in your math notebook.  $-3 \times 4 = -12$ ."
5. "Things look a little different in this next problem. Let's say we have three taken negative five times  $3 \times -5$ . How can we take a set of three negative times."
6. "For now, let's just take a set of three five times."



7. "Remember what we did before when we were working with a negative? We did the **opposite**."

8. "Let's use that strategy again."

9. Replace the green tiles with gray tiles and the solution becomes  $3 \times -5 = -15$ . Have the child record this in their math notebook.



10. "We have one other form of multiplication. Let's take multiply a set of negative two by a set of negative four."  $-2 \times -4 =$
11. "Again, we are unable to build a set of negative two negative four times. But for just this moment, let's build a set of negative two, four times."
12. "What strategy do you think we will use? Yes, we have a set of negative two taken four times and that equals negative eight."  $-2 \times 4 = -8$ .
13. "However, we need to take the set of negative two, negative four times. Let's do the **opposite**."
14. Replace the gray tiles with the green tiles producing the following product.  $-2 \times -4 = 8$
15. Have the child record this equation in their math notebook.

# Properties of Numbers

## Age

10 years

## Aim

Direct: understand the 4 properties of numbers (identity, commutative, associative, distributive)

Indirect: to use the properties in algebraic equations

## Materials

box of colored bead bars, white board, box of signs

## Presentation

1. Invite the child to the lesson.
2. "You have been working with numbers ever since you began school. You probably already know these four properties, but you may not know their names."
3. Build a set of twelve with the bead bars.
4. "What happens when I add zero to this set of bead bars? Yes, the sum will not change. This is termed the **identity property of addition**. It states that when zero is added to a set, the quantity of the set does not change."
5. Record this on the white board.
6. "What happens when I multiply this set by zero? Ah, it does change doesn't it; it becomes zero. Since any number multiplied by zero is zero, this is termed the **identity property of multiplication**."
7. Record this on the white board.
8. "Let's look at another property that you have seen."
9. Take out a five bar and an eight bar. Build the equations  $5 + 8 =$  and  $8 + 5 =$ .
10. "What do you know about these two equations? Yes, the sum will be exactly the same. This is the **commutative property of addition**. This property states that it doesn't make any difference what order the addends are in, as long as they are the same addends, they will always make the same sum."
11. "What do you think happens with multiplication. Yes, there is a **commutative property of multiplication** as well. Why do you think this is true? Yes, since multiplication is repeated addition, and addition is commutative, multiplication must be commutative as well."
12. Record this on the white board.
13. "The **associative property of addition** states that it does not matter how the addends are grouped, their sum will be the same. This property also works for multiplication for the same reason that it works for the commutative property." Build  $(2 \times 3) \times 3 =$  and  $2 \times (3 \times 3) =$ .
14. "Finally, we have the **distributive property**. Build the following  $2(3 + 4) =$  .
15. "We can either add what is in the parenthesis first, and then multiply  $(3 + 4) = 7$ .  $7 \times 2 = 14$  or we can multiply  $(2 \times 3) + (2 \times 4) =$ .  $2 \times 3 = 6$ ,  $2 \times 4 = 8$ .  $6 + 8 = 14$ . Either way we get the same answer."
16. Record this on the white board.

# Hindu Square Root

## Age

10 years

## Aim

Direct: to learn an alternative form to extract a square root

Indirect: to gain facility with mental math

## Materials

white board

## Presentation - Binomial

1. Invite the child to the lesson.
2. "As we have discussed before, mathematicians often find a faster way to do things. Different civilizations were able to extract the square root of numbers, without using algebraic notation."
3. "This method was used by the Hindus and is a process of estimating and dividing. Let's start with this problem."

$$\sqrt{625}$$

4. "You already know how to break it into periods. When you do that you can see that the root will be a two digit number."

$$\sqrt{6\text{ }25}$$

5. "Looking at the first period on the left, what is the largest square the can be extracted? Yes, we know that  $2^2$  will equal 4 and  $3^2$  would be too much.

t     u		
2		
$\sqrt{625}$	$\textcircled{2^2} = 4$	Multiply the $2^2$ to get 4 and subtract. Bring down the next period.
-4 ↓ ↓	$2(2 \times 10) = 40$ $40 + \underline{\hspace{1cm}}$	
225		Double the 2 in the ten's place to get 4. Since it is in the tens place we multiply by 10 and now have 40.

t     u		
2   5		
$\sqrt{625}$	$\textcircled{2^2} = 4$	
-4 ↓ ↓	$2(2 \times 10) = 40$ $40 + 5 = 45$	
225	$45 \times 5 = 225$	What number, when added to the 40 in the unit's place and also used as a multiplier will give a product to subtract from 225?
-225		
0		

## Presentation - Trinomial

1. Invite the child to the lesson.
2. Record the problem and break it into periods.

$$\sqrt{1\ 56\ 25}$$

h t u  
1

$$\begin{array}{r} \sqrt{1\ 56\ 25} \\ -1 \\ \hline 56 \end{array}$$

Find the largest square and subtract.  
Bring down the next period.

h t u  
1 2

$$\begin{array}{r} \sqrt{1\ 56\ 25} \\ -1 \\ \hline 56 \\ -50 \\ \hline \end{array}$$

Double the 1 to get 2 and multiply by 10 to get 20. What number, when added to the 20 in the unit's place and also used as a multiplier will give a product to subtract from 56?  $20 + 2 = 22$ .  $22 \times 2 = 44$

h t u  
1 2 5

$$\begin{array}{r} \sqrt{1\ 56\ 25} \\ -1 \\ \hline 56 \\ -44 \\ \hline 1225 \\ -1225 \\ \hline 0 \end{array}$$

Subtract and bring down the next period. Double the 12 to get 24 and multiply by 10 to get 240. What number, when added to the 240 in the unit's place and also used as a multiplier will give a product to subtract from 625?  $240 + 5 = 245$ .  $245 \times 5 = 1225$

## Follow Up

1. Special Cases
2. Cubing

# Inequalities

## Age

11 years

## Aim

Direct: to learn the symbols used in graphing inequalities on a number line

Indirect: to use previous learning in solving equations for inequalities

## Materials

white board





## Presentation 1

1. Invite the child to the lesson.
2. "Today, we will be working on a number line with some new symbols."
3. Introduce the following inequality and graphing inequality symbols with their meaning.

### *Inequality Symbols*

$<$	less than
$\leq$	less than or equal to
$>$	more than
$\geq$	more than or equal to

### *Graphing Inequality Symbols*

	less than
	less than or equal to
	more than
	more than or equal to

4. "Let's use these symbols to graph the following equation."  $x > -1$
5. "What is the least value that we must have on our number line?" (-1)
6. "In what direction will the number line progress? (to the positive)"
7. Draw the following with the correct symbol.





## Presentation 2

1. Invite the child to the lesson.
2. "We will use what we know about solving an equation to also solve an inequality."
3. Record the following equation on the white board.  $-4 + x > -2$ .
4. "What are our three rules? In order to isolate our x on one side of the equation we need to add a positive four to cancel out the negative 4. Whatever we do to one side of the equation we have to do to the other side."

$$\begin{array}{r} -4 + x > -2 \\ +4 \quad \quad +4 \\ \hline x > 2 \end{array}$$

## Follow Up

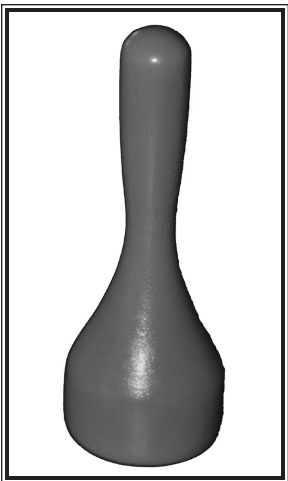
1. Solving for greater/less than or equal to inequalities

# Supplemental Pieces for Operations with Integers.

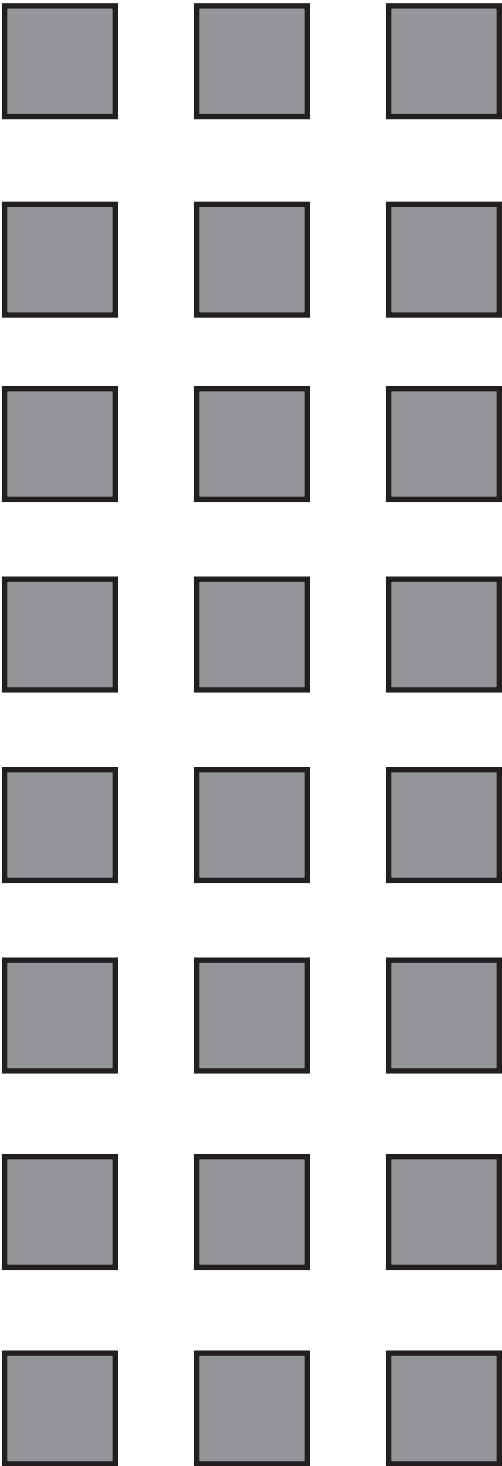
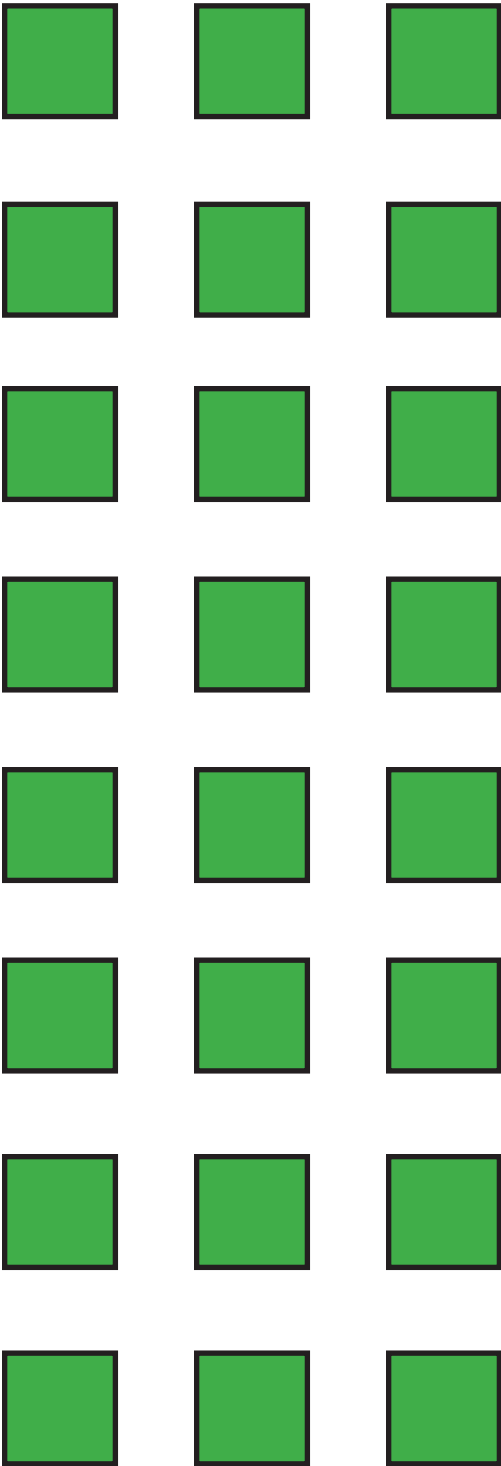
Cut out each skittle along the frame for use with division.



Cut out each skittle along the frame for use with division.



Cut out each green and gray tile along the frame. Glue a green tile and gray tile back to back, making a two sided tile.



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