Working with Negative Numbers

ETC MONTESSORI

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Core Standards

6.NS.C.5

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.C.6

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

7.NS.A.1

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

- Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

- Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts.
- Apply properties of operations as strategies to multiply and divide rational numbers.
- Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats

7.EE.B.3

olve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

HSA-APR.D.7

Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Historical Background

Although the first set of rules for dealing with negative numbers was stated in the 7th century by the Indian mathematician Brahmagupta, it is surprising that in 1758 the British mathematician Francis Maseres was claiming that negative numbers "... darken the very whole doctrines of the equations and make dark of the things which are in their nature excessively obvious and simple" .

Maseres and his contemporary, William Friend took the view that negative numbers did not exist. However, other mathematicians around the same time had decided that negative numbers could be used as long as they had been eliminated during the calculations where they appeared.

It was not until the 19th century when British mathematicians like De Morgan, Peacock, and others, began to investigate the 'laws of arithmetic' in terms of logical definitions that the problem of negative numbers was finally sorted out.

In 200 BCE the Chinese number rod system (see note1 below) represented positive numbers in Red and Negative numbers in black. An article describing this system can be found here . These were used for commercial and tax calculations where the black canceled out the red. The amount sold was positive (because of receiving money) and the amount spent in purchasing something was negative (because of paying out); so a money balance was positive, and a deficit negative.

The concept also appeared in Astronomy where the ideas of 'strong' and 'weak' were used for approximating a number from above or below. For example approaching 5 from above means for example, starting with 5.2 you can find better approximations 5.1, 5.05, 5.025. Thus 5.025 was called a 'strong' approximation and a number like 4.9 'weak'. So 'strong' numbers were called positive and 'weak' numbers negative

In India , negative numbers did not appear until about 620 CE in the work of Brahmagupta (598 - 670) who used the ideas of 'fortunes' and 'debts' for positive and negative. By this time a system based on place-value was established in India, with zero being used in the Indian number system. Brahmagupta used a special sign for negatives and stated the rules for dealing with positive and negative quantities as follows:

A debt minus zero is a debt.
A fortune minus zero is a fortune.
Zero minus zero is a zero.
A debt subtracted from zero is a fortune.
A fortune subtracted from zero is a debt.
The product of zero multiplied by a debt or fortune is zero.
The product of zero multiplied by zero is zero.
The product or quotient of two fortunes is one fortune.
The product or quotient of two debts is one fortune.
The product or quotient of a debt and a fortune is a debt.
The product or quotient of a fortune and a debt is a debt.

The conflict between geometry and algebra

The ancient Greeks did not really address the problem of negative numbers, because their mathematics was founded on geometrical ideas. Lengths, areas, and volumes resulting from geometrical constructions necessarily all had to be positive. Their proofs consisted of logical arguments based on the idea of magnitude. Magnitudes were represented by a line or an area, and not by a number (like 4.3 meters or 26.5 cubic centimeters). In this way they could deal with 'awkward' numbers like square roots by representing them as a line. For example, you can draw the diagonal of a square without having to measure it (see note 2 below).

About 300 CE, the Alexandrian mathematician Diophantus (200 - c.284 CE) wrote his *Arithmetica* , a collection of problems where he developed a series of symbols to represent the 'unknown' in a problem, and powers of numbers. He dealt with what we now call linear and quadratic equations. In one problem Diophantus wrote the equivalent of $4 = 4x + 20$ which would give a negative result, and he called this result 'absurd'.

In the 9th century in Baghdad Al - Khwarizmi (c.780 - c.850 CE) presented six standard forms for linear or quadratic equations and produced solutions using algebraic methods and geometrical diagrams. In his algebraic methods he acknowledged that he derived ideas from the work of Brahmagupta and therefore was happy with the notion of negative numbers. However, his geometrical models (based on the work of Greek mathematicians) persuaded him that negative results were meaningless (how can you have a negative square?). In a separate treatise on the laws of inheritance, Al-Khwarizmi represents negative quantities as debts.

In the 10th century Abul -Wafa (940-998 CE) used negative numbers to represent a debt in his work on 'what is necessary from the science of arithmetic for scribes and businessmen'?. This seems to be the only place where negative numbers have been found in medieval Arabic mathematics. Abul-Wafa gives a general rule and gives a special case where subtraction of 5 from 3 gives a "debt" of 2. He then multiples this by 10 to obtain a "debt" of 20, which when added to a 'fortune' of 35 gives 15.

In the 12th century Al - Samawal (1130 - 1180) had produced an algebra where he stated that:

if we subtract a positive number from an 'empty power', the same negative number remains,
if we subtract the negative number from an 'empty power', the same positive number remains, the product of a negative number by a positive number is negative, and by a negative number is positive.
Negative numbers did not begin to appear in Europe until the 15th century when scholars began to study and translate the ancient texts that had been recovered from Islamic and Byzantine sources. This began a process of building on ideas that had gone before, and the major spur to the development in mathematics was the problem of solving quadratic and cubic equations.

As we have seen, practical applications of mathematics often motivate new ideas and the negative number concept was kept alive as a useful device by the Franciscan friar Luca Pacioli (1445 - 1517) in his Summa published in 1494, where he is credited with inventing double entry book-keeping.

Solving equations

The story of the solution of equations begins in Italy in the 16th century (see note 3 below). This story is full of intrigue and deception because methods of solution were kept secret. The issue which caused most consternation at the time was the meaning of $\sqrt{-1}$. In fact, Cardano (1501 - 1576) in his Ars Magna of 1545 had to solve a problem where $\sqrt{-15}$ appeared.

Cardano found a sensible answer (see note 4 below) by working through the algorithm, but he called these numbers ‘ficticious’ because not only did they disappear during the calculation, but they did not seem to have any real meaning. However, by 1572, the Italian engineer, Bombelli (1526 - 1572) had provided the correct rules for working with these ‘imaginary’ numbers(see note 5 below).

In the 17th and 18th century, while they might not have been comfortable with their ‘meaning’ many mathematicians were routinely working with negative and imaginary numbers in the theory of equations and in the development of the calculus.

The English mathematician, John Wallis (1616 - 1703) is credited with giving some meaning to negative numbers by inventing the number line, and in the early 18th century a controversy ensued between Leibniz, Johan Bernoulli, Euler and d’Alembert about whether $\log(-x)$ was the same as $\mathrm{Log}(x)$.Wallis

By the beginning of the 19th century Caspar Wessel (1745 - 1818) and Jean Argand (1768 - 1822) had produced different mathematical representations of ‘imaginary’numbers, and around the same time Augustus De Morgan (1806 - 1871), George Peacock (1791 - 1858) William Hamilton (1805 - 1865) and others began to work on the ‘logic’of arithmetic and algebra and a clearer definition of negative numbers, imaginary quantities, and the nature of the operations on them began to emerge.

Negative numbers and imaginaries are now built into the mathematical models of the physical world of science, engineering and the commercial world.

There are many applications of negative numbers today in banking, commodity markets, electrical engineering, and anywhere we use a frame of reference as in coordinate geometry, or relativity theory.

Vocabulary

Integer - Any whole number that is not a fraction. It includes the number 0 as well as all the positive and negative numbers.

Positive Integer - Any number greater than 0. They are usually used for counting.

Negative Integer - Any number less than 0. Negative integers represent the opposite of real numbers.

Absolute Value - The value of a real number regardless of its relation to other values or the sign it has.

Addend - The quantity of something that is added to another quantity.

Sum - The total quantity of two addends or two other quantities.

Minuend - The quantity of a number from which another quantity is removed.

Subtrahend - The quantity of a number that is to be subtracted from another quantity.

Difference - The quantity by which two other numbers differ.

Divisor - A number by which another number is to be divided by.

Dividend - The amount indicating the quantity or value that is to be distributed.

Quotient - The value of the result obtained by dividing one number by another.

Remainder - the number which is left over in a division.

Multiplicand - a quantity which is to be multiplied by another.

Multiplier - The quantity by which a different number is to be multiplied

Product - The result of multiplying two quantities.

Answer Key

The emphasis is placed on understanding of the concept rather than solving a great deal of problems.

Activity #1

Answers will vary based on analysis level and skills.

Activity #2

Research Project. Answers will vary.

Activity #3

Answers will vary. However, the number line shown and the number line found on a coordinate plane do not differ. A number line is a number line. The coordinate plane is made up of two number lines. One horizontal and a second that is vertical. They meet at the point of origin which is (0,0)

Activity #4

By definition there is a -0, just like there is a +0. However, the conclusion we want the students to reach is that both of these numbers exist, or reside on the same point on a number line. 0 is a very unique number since it has the following properties:

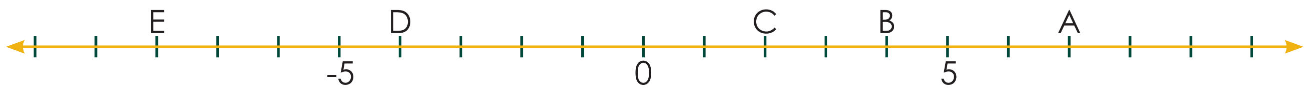
1. Any number added to 0 will result in itself. Regardless of whether the 0 is positive or negative.
2. Any number multiplied by 0 will be distributed 0 times therefore it is equal to 0.
3. Any number divided by 0 is undefined
4. 0 divided by any number will equal to 0

Activity #5

The absolute value is the magnitude of a number regardless of it being positive or negative:

5. |20| = 20
6. |-16| = 16
7. |-5| = 5
8. |12| =12
9. The absolute value of -7 = 7
10. The absolute value of 6 = 6
11. The absolute value of -3 = 3
12. |0| = 0
13. |-1,000| = 1,000
14. -|-15| = -15
15. -|15| = -15
16. |-23| = 23

Activity #6



Greater than negative 4, but less than positive 3 : C
Less than eight, but greater than negative two : answers will vary. A, B, C are possible.
Less than or equal to positive two, but greater than negative one : C
Greater than negative seven, but larger than negative four : D, C are possible
Less than negative nine, and less than zero : E, D are possible
Greater than negative one but less than positive five : C and B are possible

Activity #7

-8 ___ 0 <
|-8| ___ |20| <
4 ___ -13 >
|-5| ___ |-13| <
-10 ___ -3 <
|10| ___ |-10| =
9 ___ |-17| <
12 ___ -2 >
1 ___ 0 >
16 ___ |-16| =

Activity #8

The idea here is to get the students to understand that the scale on a number can differ. The distance between the points can be equal to 10 or 20 or even 30 points. They must be consistent with the division but they can decide what the distance is symbolized as.

The coldest state would be Alaska since it is -80

Activity #9

If we were to symbolize the surface of the water as 0 then anything above the water would be considered positive. Anything below the water would be considered negative. Therefore, Nitsch would be -215 and Streeter would be -116.

Activity # 10

The tiles used would be gray for the negative numbers and green for the positive numbers. Exercises 1-12 are simply counting the correct number of tiles with the correct color.

Second set:

The goal is to get the students to understand that green tiles and gray tiles can cancel each other out. So, 2 green tiles can cancel 2 gray tiles resulting in an answer of 0. With this in mind:

1. +3	2. +2	3. 0	4. +6	5. +1	6. -2	7. +4
8. -2	9. +4	10. +2	11. +8	12. -6		

Activity # 11

a. +9 b. -9 c. +7 d. -4 e. -12

Activity # 12

a. +4 b. +2 c. -2 d. 0 e. -8

Activity # 13

a. -2 b. -6 c. -2 d. -8 e. -5

Activity # 14

a. +8 b. +4 c. +12 d. +12 e. +19

Activity # 15

a. -10 b. +6 c. +7 d. -12 e. +37

Activity # 16

1. -21	2. -53	3. -67	4. Answers will vary.

Activity # 17

1. 4	2. 24	3. 19	4. 6	5. 4 r 4	6. 6 r 1	7. 6 r 2
8. 6 r 3	9. 2 r 3	10. 2 r 8				

Activity # 18

1. -3 r 6	2. -5 r 5	3. -12	4. -3 r 3	5. -5 r 2	6. -12 r 1	7. -6 r 1
8. -7 r 1	9. -3 r 3	10. -2 r 7				

Activity # 19

1. -5	2. -5	3. -8	4. -2 r 1	5. -6 r 2	6. -3	7. -7 r 1
8. -4	9. -9	10. -1				

Activity # 20

1. 5	2. 5	3. 8	4. 2 r 1	5. 6 r 2	6. 3	7. 7 r 1
8. 4	9. 9	10. 1				

Activity # 21

1. -1	2. -18	3. -8	4. -10	5. -7	6. -6	7. 9 r 7
8. -3	9. +20 r 3	10. +8	11. +14	12. -15	13. -8	14. -14
15. +4	16. +5	17 -1	18. -7			

Activity # 22

1. 48	2. 160	3. 20	4. 21	5. 12	6. 36	7. 20

Activity # 23

1. -180	2. -245	3. -72	4. -136	5. -160	6. -147	7. -66
8. -324	9. -36	10. -16				

Activity # 24

1. -180	2. -245	3. -72	4. -136	5. -160	6. -147	7. -66
8. -324	9. -36	10. -16				

Activity # 25

1. +180	2. +245	3. +72	4. +136	5. +160	6. +147	7. +66
8. +324	9. +36	10. +16				

Activity # 26

1. -576	2. -18	3. -392	4. -1440	5. -448	6. -864	7. +632
8. -363	9. +738	10. +288	11. +244	12. -15	13. -3375	14. -56
15. +196	16. +125	17 -1	18. -7			

ETC Montessori®

979 Reseda Dr. • Houston • TX • 77062

T: 877 409 2929 • F: 877 409 7402
Intl: 281 984 7213

ETCmontessori.com