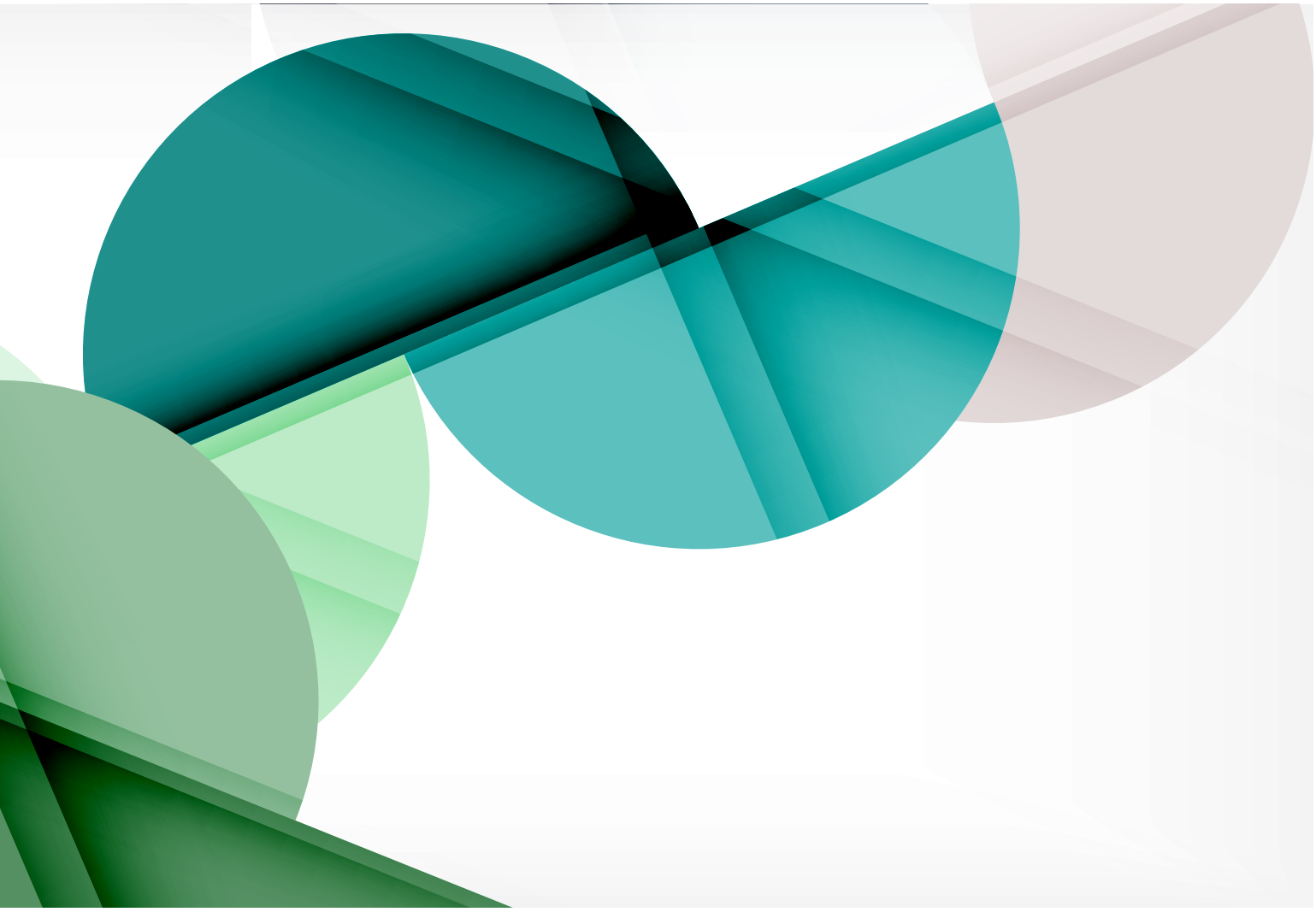
Geometry of a Circle

ETC
MONTESSORI

Tale of Contents



COMMON CORE

4.G.A.1
- Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

5.G.B.3
- Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

5.G.B.4
- Classify two-dimensional figures in a hierarchy based on properties.

7.G.A.1
- Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

7.G.A.2
- Draw construct, and describe geometrical figures and describe the relationships between them.

7.G.A.3
- Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

7.G.B.4
- Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.B.6
- Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

8.G.A.1
- Verify experimentally the properties of rotations, reflections, and translations

8.G.A.2
- Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.3
- Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.A.4
- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

HSG-GMD.A.1
- Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

ANSWER KEY

AREA OF A CIRCLE

Card 1A

r = 2.56 in or 6.55 cm
d = 5.12 in or 13.1 cm
C = 16.07 in or 41.13 cm
A = 20.56 in² or 134.7 cm²

Card 1B

A = 8.748 m²

Card 2A

A = 143.06 m²

r = 1.5 in.
A = 7.06 in²

Card 2B

r = 7.mm
A = 153.86 mm²

A = 2205.06 mm²

Card 3A

Use formula (a)

Card 3B

Use formula $A = \frac{C \times r}{2}$

C = 12 ft²

Card 4A

Area of a cooler top may be found using formula $A = \pi r^2$. This yields an area of 8.72 in²
Therefore, she can fit 4 coolers in the van.

Card 4B

1. A = 1,639.53 m²
2. A = 113 cm²
3. A = 3.14 in²
4. A= 92.86 ft²
5. A = 23.98 in²
6. A = 1.898.08 cm²
7. A = 8965.15 mm²

Card 5A

r = 3.2 m

A = 32.17 m²

A total of 16 kids can fit around the circle.

Card 5B

Mathematically you can find the circumference of ⊙K and the diameter of ⊙A. (this is a close approximation since student has not studied segments yet).

Empirically you can draw out ⊙K and then take c ⊙A and move its center around the circumference.

An interesting observation is if the center of ⊙A is on the circumference or if it is found outside the circumference.

AREA OF A SECTOR

Card 1A

The idea is to find the entire area of a cookie by using $A = \pi r^2$ and then dividing it in half.

Therefore, A = 19.625 in² and the area of the cookie that was eaten is 9.8125 in²

Card 1B

Answers will vary

Card 2A

The formula used will be

$$Area\ of\ Sector = \frac{\theta}{360}\pi r^2$$

Given:

Minor Sector:

θ = 120°
r=11.5
A= 138.28

Major Sector:

θ = 240°
r=11.5
A= 276.56

Card 2B

d=18 in
r=9 in

With the information given we can calculate the area of one piece and find the area of pizza that each individual has consumed.

Marcus (5 pieces) = 158.95 in²
Jen (3 pieces) = 95.37 in²
Bobby (2 pieces) = 63.58 in²
Heather (3 pieces) = 95.37 in²
Justin (3 pieces) = 95.37 in²
Shannon (1 piece) = 31.79 in²
Becka (4 pieces) = 127.16 in²
Shelly (2 pieces) = 63.58 in²
Lisa (1 piece) = 31.79 in²

Card 3A

The diameter of the cookie cake is d = 10 in
θ = 22.5° and r = 5 in.

If we start with the formula $A = \frac{(r \times L)}{2}$, then we can find L = 1.96 in.

So, to find the area we substitute 1.96 for L and we find that A= 4.90 in²

Card 3B

Given:

d = 10 in
r = 5 in
θ = 45°

Then we can calculate:

Sector Area = 9.82 in²
Brenda had 2 so she had a total of 19.64 in²
If the circle has a surface area of 78.5 in² that means there are 58.86 in² left over.

Card 4A

Given:

d = 10.75 Km
r = 5.375 Km
θ = 180°

Therefore, L = 16.9 Km
A= 45.4 Km²

Card 4B

a. 71.4°
b. 60.6°
c.45.5°
d.26°
e.125.6°
f. 41.1°

Card 5A

Answers will vary.

Card 5B

A = 5.024 ft²
θ = 36°
r = ? (3.998 f t)
d = 7.997 ft

AREA OF A SEGMENT

Card 1A

Answers will vary depending on the accuracy of measurements. However, the following is an acceptable range:

r = .8773
θ = 153°
h = .672
Arc = 2.343
Chord = 1.706
Area = .853

Card 1B

Answers will vary depending on the accuracy of measurements. However, the following is an acceptable range:

r = 2.5625
θ = 124°
$A_{rc}L$ = 5.55
Chord = 4.525
Segment Area = 4.38 in²

Card 2A

Answers will vary

Card 2B

θ = 120°
A_{seg} = 49.7 in²

Card 3A

Blue = 1.312 in²
Orange = 1.53 in²
Gray = .31 in²

Card 3B

Answers will vary

Card 4A

Answers will vary based on accuracy of measurements. However, answers should fall near the following acceptable range.

$\overline{CD}$ = 1.185 cm (h)
$\overline{AB}$ = 3 cm (K)
$\overline{AC}$ = 2 cm (r)
$\overset{\frown}{AB}$ = 1/3 of the circumference (L)
or 4.186
Therefore, C = 12.56

A_{major} = 12.56 - 2.405
A_{major} = 10.155

Card 4B

Answers will vary based on accuracy of measurements. However, answers should fall near the following acceptable range.

h = 23 mm
r = 32 mm
K = 45 mm
C = 201.061 mm
therefore
A_{circle} = 3215.36 mm²
A_{major} = 2928.7 mm²
A_{minor} = 286.66 mm²

Card 4B

Answers will vary based on accuracy of measurements. However, answers should fall near the following acceptable range.

r = 2 cm
L = 3.14 cm
K = 2.3 cm
h = 1.3 cm
θ = 90°
C_{minor} = 3.14 cm

A_{minor} = 1.643 cm2

Card 5B

Answers will vary based on accuracy of measurements. However, answers should fall near the following acceptable range.

If
r= 2.25 in.
Then the side of the triangle will be 3.125 in.

the area of the square is 9.765 in². If we subtract the area of the square from the area of the circle then the area of all minor segments is 6.131 in² and since we have 4 of these segments which are equal since the square has equal sides, then each segment has an area of 1.532 in²

AREA OF AN ANNULUS

Card 1A

Answers will vary depending on the accuracy of measurements. However, the following is an acceptable range:

$A_{annulus}$ = 13.969

Card 1B

Answers will vary depending on the accuracy of measurements. However, the following is an acceptable range:

R = 28
r = 17

$$A_{annulus} = \pi(R^2 - r^2)$$

Aannulus = 1554.3 cm²

Card 2A

Answers will vary depending on the accuracy of measurements. However, the following is an acceptable range:

To solve you will need to find the following:

1. Find the annulus of circles U and S
2. Find the annulus of circles S and B
3. Find the annulus of circles B and F

Card 2B

$$A_{annulus} = \pi(R^2 - r^2)$$
$$188.4 = 3.14(R^2 - r^2) \implies$$
$$188.4 = 3.14R^2 - 12.56 \implies$$
$$188.4 + 12.56 = 3.14R^2 \implies$$
$$200.96 = 3.14R^2 \implies$$
$$\frac{200.96}{3.14} = R^2 \implies$$
$$64 = R^2 \implies$$
$$\sqrt{64} = R \implies$$
$$8 = R$$

Card 3A

R = 2.375
r = .745
A = 15.998 in²

Card 3B

Cans:
A = 87.823 in²

Plastics:
A = 135.881 in²

Paper:
A = 628.318 in²

Card 4B

The actual structure looks like the picture below:

a. There are 4 circles. However, should extend the outermost area into a circle then there are 5 circles.
b. Area of pool is 3848.45 fm², Area of grass is 1178.10 m², Area of walking path is 1621.06 m², area of Stone structure is 6902 m2

Card 5A

R = 2.25 cm
r = .875
Aannulus 13.499 cm²

Card 5B

If the size is 72 mm then A = 3976.470 mm²
If the size is 82 mm then A = 5185.984 mm²

ARC OF A CIRCLE

Card 1A

1. Answers will vary based on letters chosen.

Card 1B

First angle = 45º
Second angle = 90º
Third angle = 125º
Fourth angle = 270º
No, the angle measurements would not be different.

Card 2A

1. 340º
2. Drawings will be constructed by students using their measurement tools. Answers and drawings will vary.
3. They will be congruent
4. They will be congruent

Card 2B

1. Student should trace the circle and show center.
2. Answers will vary.
3. The central angels will be drawn based on the radius of the circle. Two radii will extend from the center to each of the endpoints of the chord. Answers will vary based on the individual design.
4. Answers will vary based on the individual design.
5. Answers will vary based on the individual design.
6. Answers will vary based on the individual design.
7. In the same circle, or in congruent circles, two minor arcs are congruent if and only if their chords are congruent.

Card 3A

1. Answers will vary based on the individual design.
2. Answers will vary based on the individual design.
3. It increases.
4. When the length of $\bar{PD}$ is decreased then $\overset{\frown}{l}$ increases. In general when the chord increases then the arc will decrease and vise versa.
5. If two chords of a circle are unequal in length, then the longer chord is nearer to the center of the circle.
6. If two chords of a circle are not equidistant

from the center, then the longer chord is nearer to the center of the circle.

Card 3B

1. False. A diameter is the longest chord of a circle.
2. True. RS>PQ. If two chords of a circle are unequal in length, then the longer chord is nearer to the center of the circle.
3. True. If a diameter is perpendicular to the chord, then it bisects the chord and its arc.

TANGENTS OF A CIRCLE

Card 1A

1. Answers will vary.
2. Answers will vary.
3. Infinite number of lines can pass through the point. However, only one line can pass through the point and be tangent to the circle.
4. Answers will vary.
5. Answers will vary.

Card 1B

1. Answers will vary based on accuracy of measurement.
2. 90º
3. If a line is tangent to a circle, then the line is perpendicular to the radius of the point of tangency.

Card 2A

Answers will vary.

Card 2B

Answers will vary.

Card 3A

It's the area of total eclipse.

Card 3B

1. Perimeter is 72.
2. Right triangle
3. Based on the Pythagorean Theorem. $BC^2 + CA^2 = AB^2$

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