Pre-Algebra

Properties of Real Numbers

Identity Properties

Addition: $a + 0 = a$ and $0 + a = a$

Multiplication: $a \times 1 = a$ and $1 \times a = a$

Commutative Properties

Addition: $a + b = b + a$

Multiplication: $a \times b = b \times a$

Associative Properties

Addition: $(a + b) + c = a + (b + c)$

Multiplication: $(a \times b) \times c = a \times (b \times c)$

Inverse Properties

Addition:
$a + (-a) = 0$ and $-a + a = 0$

Multiplication:
$a \times 1 = 1$ and $1 \times a = 1 (a \neq 0)$

Distributive Properties

$a(b + c) = ab + ac$ $\qquad (b + c)a = ba + ca$

$a(b - c) = ab - ac$ $\qquad (b - c)a = ba - ca$

Properties of Equality

Addition: If $a = b$, then $a + c = b + c$
Subtraction: If $a = b$, then $a - c = b - c$
Multiplication: If $a = b$, then $a \times c = b \times c$
Division: If $a = b$, and $c \neq 0$, then $\dfrac{a}{c} = \dfrac{b}{c}$

Substitution: If $a = b$, then b can replace a in any expression
Reflexive: $a = a$
Symmetric: If $a = b$, then $b = a$
Transitive: If $a = b$ and $b = c$, then $a = c$

Cross Product Property

$\dfrac{a}{b} = \dfrac{c}{d}$ is equivalent to $ad = bc$

Zero-Product Property

if $ab = 0$ then $a = 0$ or $b = 0$

Closure Property

$a + b$ is a unique real number.

$a \times b$ is a unique real number.

Density Property

Between any two rational numbers, there is at least one other rational number.

Properties of Inequality

Addition: If $a > b$, then $a + c > b + c$.
 If $a < b$, then $a + c < b + c$.
Subtraction: If $a > b$, then $a - c > b - c$.
 If $a < b$, then $a - c < b - c$.

Multiplication:
If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c < 0$, then $ac < bc$.
If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c < 0$, then $ac < bc$

Division:
If $a > b$ and $c > 0$, then $\dfrac{a}{c} > \dfrac{b}{c}$.

If $a < b$ and $c < 0$, then $\dfrac{a}{c} < \dfrac{b}{c}$.

If $a > b$ and $c > 0$, then $\dfrac{a}{c} > \dfrac{b}{c}$.

If $a < b$ and $c < 0$, then $\dfrac{a}{c} < \dfrac{b}{c}$

Transitive If $a > b$ and $b > c$, then $a > c$.
Comparison: If $a = b + c$ and $c > 0$, then $a > b$.

Properties of Exponents

For any nonzero number a and any integers m and n:

Zero Exponent: $\qquad a^0 = 1$
Negative Exponent $\qquad a^{-n} = \dfrac{1}{a^n}$

Product of Powers: $\qquad a^m \times a^n = a^{m+n}$

Quotient of Powers: $\qquad \dfrac{a^m}{a^n} = a^{m-n}$

Pre-Algebra Sequence

- Teacher's Answer Key
- Nomenclature (color coded with Activity cards) Present nomenclature as an introduction to the specific concept
- Activity Cards with the following concepts:
  - Algebraic Expressions and Integers
  - Properties of Numbers
  - Fractions
  - Factors
  - Exponents
  - Solving Equations
  - Solving Inequalities
  - Solving Multi-step Equations
  - Solving Multi-step Inequalities, and
  - Patterns and Polynomials
- Control Chart depicting Properties of Numbers

Answer Key

Algebraic Expression and Integers

Card #

2   **1)** Variable Expression; y   **2)** Numerical Expression   **3)** Variable Expression; a

   4) Variable Expression; p   **5)** Numerical Expression   **6)** (6+5+8)7 or 6(7) +5(7) +8(7)

3   **1)** 28   **2)** 15   **3)** 14y   **4)** 0   **5)** 9   **6)** Answer should be 2 by order of operations. Jake did
                                                        subtraction before multiplying

4   **1)** 7   **2)** 9   **3)** 6   **4)** 25   **5)** 120

5   **1)** 16   **2)** 16   **3)** 8   **4)** 15   **5)** 2   **6)** 8

6   **Positive, Negative**

7   **1)** 68   **2)** 1   **3)** 10   **4)** -4x-5   **5)** 40   **6)** 3   **7)** 12   **8)** 10

8   **1)** 32   **2)** 1   **3)** 1.875   **4)** 36   **5)** 189   **6)** △

9   **1)** -6   **2)** -3   **3)** -3   **4)** 4   **5)** 8   **6)** Answer is actually 24

Properties of Numbers

Card #

1   **1)** 11=11   **2)** 73=73   **3)** 30=30   **4)** 240=240   **5)** 80=80   **6)** 1500=1500

2   **1)** 12=12   **2)** 24=24   **3)** 13=13   **4)** 64=64   **5)** -60=-60   **6)** -36=-36

3   **1)** 356   **2)** 11   **3)** 4   **4)** 4   **5)** 8   **6)** -5

5   **1)** 5(4)-5(2) = 20 **2)** 8(3) + 8(6) =72 **3)** -20(9)-(-20)(4) = -100   **4)** 13(6) + 13(-5) = 13
           5) (-7) (5) - (-7) (9) = 28   **6)** 12 (-3) - 12 (-8) = -132   **7)** 9(-8) + 9(5)= -27
           Yes, the same results are produced. According to the Commutative properties of multiplication and
            addition, the addition can be done first then multiplied, or the multiplication can be done first and then
            added.

5   **1)** 9+13u   **2)** x+16   **3)** -2x-2c or -2(x + c)   **4)** 6h-8v+8   **5)** 11a + 5b + c + 5
           6) 14f+22p   **7)** 2a + 2b + 2c + 2d or 2(a+ b+ c + d)

Fractions, Factors, and Exponents

Card #

1

	1)	2)	3)	4)	5)	6)	7)	8)
2	No	No	No	Yes	Yes	Yes	No	Yes
3	Yes	Yes	Yes	Yes	Yes	Yes	No	No
5	Yes	No	No	No	No	Yes	Yes	Yes
9	Yes	No	No	No	Yes	Yes	No	No
10	No	No	No	No	No	Yes	No	Yes

2   **1)** $5^7 = 78,125$ **2)** $3^3 = 27$ **3)** $6^4 = 1296$ **4)** $14^2 = 196$ **5)** $10^5 = 100,000$ **6)** $2^6 = 64$

3   **1)** Prime   **2)** Composite   **3)** Prime   **4)** Composite   **5)** Composite   **6)** Composite

     7) Prime   **8)** Composite   **9)** Prime

4   **1)** $58 = 2 \cdot 29$ **2)** $222 = 2 \cdot 3 \cdot 37$ **3)** $36 = 2 \cdot 2 \cdot 3 \cdot 3$ **4)** $108 = 3 \cdot 3 \cdot 3 \cdot 2 \cdot 2$

     5) $450 = 5 \cdot 5 \cdot 3 \cdot 3 \cdot 2$ **6)** $37 = 1 \cdot 37$ **7)** $900 = 5 \cdot 5 \cdot 3 \cdot 3 \cdot 2 \cdot 2$ **8)** $144 = 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

     9) $256 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

5   **1)** 5   **2)** 8   **3)** 13   **4)** 4   **5)** 9   **6)** 7   **7)** 10   **8)** 8   **9)** 26

6   **1)** Yes   **2)** Yes   **3)** Yes   **4)** No   **5)** Yes   **6)** No

7   **1)** $5^9 = 1,953,125$ **2)** $6^{10} = 60,466,176$ **3)** $1^5 = 1$ **4)** $4^{12} = 16,777,216$

     5) $8^7 = 2,097,152$ **6)** $3^{14} = 4,782,969$

8   **1)** $2^{12} = 4096$ **2)** $6^6 = 46,656$ **3)** $5^8 = 390,625$ **4)** $12^2 = 144$ **5)** $20^6 = 64,000,000$

     6) $7^3 = 343$

9   **1)** $6^5 = 7776$ **2)** $2^6 = 64$ **3)** $12^2 = 144$ **4)** $15^1 = 15$ **5)** $9^2 = 81$ **6)**
$5^5 = 3125$

10   **1)** $15^{-4} = \frac{1}{15^4} = \frac{1}{50,625}$ **2)** $5^{-6} = \frac{1}{5^6} = \frac{1}{15625}$ **3)** $8^{-1} = \frac{1}{8}$ **4)** $9^{-2} = \frac{1}{9^2} - \frac{1}{81}$

     5) $2^0 = 1$ **6)** $23^{-2} = \frac{1}{23^8} = \frac{1}{529}$ **7)** $7^{-4} = \frac{1}{7^4} = \frac{1}{2401}$

11   **1)** $6.25 \cdot 10^5$ **2)** $7.3 \cdot 10^1$ **3)** $2.06 \cdot 10^2$ **4)** $1.3 \cdot 10^3$ **5)** 62,000   **6)** 3,547,000

     7) 217.1   **8)** 4.564

12 1) $3^2 \cdot 5^2 = 225$ 2) $7^0 \cdot 4^0 = 1$ 3) $6^{-3} \cdot 8^{-3} = \dfrac{1}{110,592}$ 4) $\dfrac{200^4}{20^4} = 10,000$ 5) $\dfrac{15^4}{5^4} = 81$

6) $\dfrac{6^4}{2^4} = 81$

Solving Equations and Inequalities

Card #

1 **1)** y= 2 **2)** x =6 **3)** g=13 **4)** d=0 **5)** c=1 **6)** x=0 **7)** b=2

2 **1)** $x \leq 1$ **2)** $x \leq 2$ **3)** $b \geq 1$ **4)** $b \geq -6$ **5)** $k < 9$ **6)** $z > 1$
7) $p > -3$

3 **1)** $h \leq 9$ **2)** $y \geq 5$ **3)** $c > 2$ **4)** $z > 5$ **5)** $k \geq 3$ **6)** $y \leq 2$
7) $m > 2$

Solving Multi-Step Equations and Inequalities

1   1) $x > 5$ 2) $x = 3$ 3) $y = -2$ 4) $z = 1$ 5) $y = -3$ 6) a= 3
7) $x \le 5$ 8) $x > 1$

2   1) $a = 25$ 2) $y = 18$ 3) $w = 33.\overline{4}$ 4) $k = 193$ 5) $x = 27$ 6) $z = +32$

3   1) $x = 48$ 2) $y = 8$ 3) $z = 3$ 4) $g = -200$ 5) $k = -500$ 6) $d = -112$

4   1) 4x+x = $15.35   2) $\dfrac{2\ hrs}{100\ papers} = \dfrac{X\ hrs}{260\ papers}$ 3) Let the amount that Justin and Caitlin pay be x;

   5x = $15.35   X=5.2 hours   x + x + (x + 50) = $614
   x = $3.07   3x+50=$614
                                                     3x = $564
                                                     x = $188
                                                     Justin and Caitlin paid $188 and Jenn
                                                     paid $238

5   1) c = 7   2) z = -12   3) x = 15   4) y = -5   5) x = 1   6) a = 4   7) k = 12

6   1) $y \le -3$ 2) $k \ge -17$ 3) $x > 6$ 4) $a > 3$ 5) $b > -6$ 6) $z < -160$
7) $x \ge 4$

7   1) $a = \dfrac{4b-3}{4}$ 2) $a = \dfrac{x}{27} - \dfrac{2}{3}$ 3) $a = 24y - 68$ 4) $a = 9 - 2z$ 5) $a = 4.5k + 24$

   6) $a = \dfrac{b}{30} - \dfrac{c}{6} + 3p$

Patterns and Polynomials

Card #

1   **1)** 5; 25, 30, 35   **2)** 0.25; 1, 1.25, 1.5   **3)** -4; -8, -12, -16   **4)** 6; 56, 62, 68   **5)** -15; -41, -56, -71
       6) 2; 15, 17, 19   **7)** 15; 75, 90, 105   **8)** -21; 16, -5, -26

2   **1)** ×2; 32, 64, 128 **2)** ×.5 1.25; 216, .625, .3125 **3)** ÷2; 1.75, 0.875, 0.4375   **4)** ×7; 7203, 50421,
352947 **5)** ÷-3; $-\dfrac{2}{3}, \dfrac{2}{9}, -\dfrac{2}{27}$ **6)** ÷4; 0.21875, 0.0546875, 0.0136718 **7)** ×(-1); 7, -7, 7 **8)** ÷10, 1, 0.1, 0.01

3   **1)** Polynomial, Binomial   **2)** Monomial   **3)** Polynomial, Trinomial **4)** Polynomial, Binomial
       5) Monomial   **6)** Polynomial

4   **1)** $-3x^2 - 4x + 9$ **2)** $3x^3 - 3x^2 + 4x - 5$ **3)** $10x^4 + 11x^3 - 5x^2$ **4)** $4x^2 + 4x + 7$
       5) $4x^3 - x^2 + 9x - 12$

5   **1)** $5x^3 + 2.5x^2 - 15x$ **2)** $-16x^2 - 32x + 40$ **3)** $-6x^7 - 12x^5 + 7x^2 + 2x + 6$
       4) $4x^5 + 4x^4 + 4x^3 - 7x^2 + 4$ **5)** $7x^4 - 8x^3 + 22x^2 - 92x - 60$ **6)** $3x^4 - 18x^3 + 24x - 3$

6   **1)** $8x^2 - 14x - 30$ **2)** $5x^2 - 14x - 3$ **3)** $63x^2 + 31x - 10$ **4)** $24x^2 - 90x - 24$
       5) $48x^2 + 4x - 24$ **6)** $108x^2 + 111x - 184$