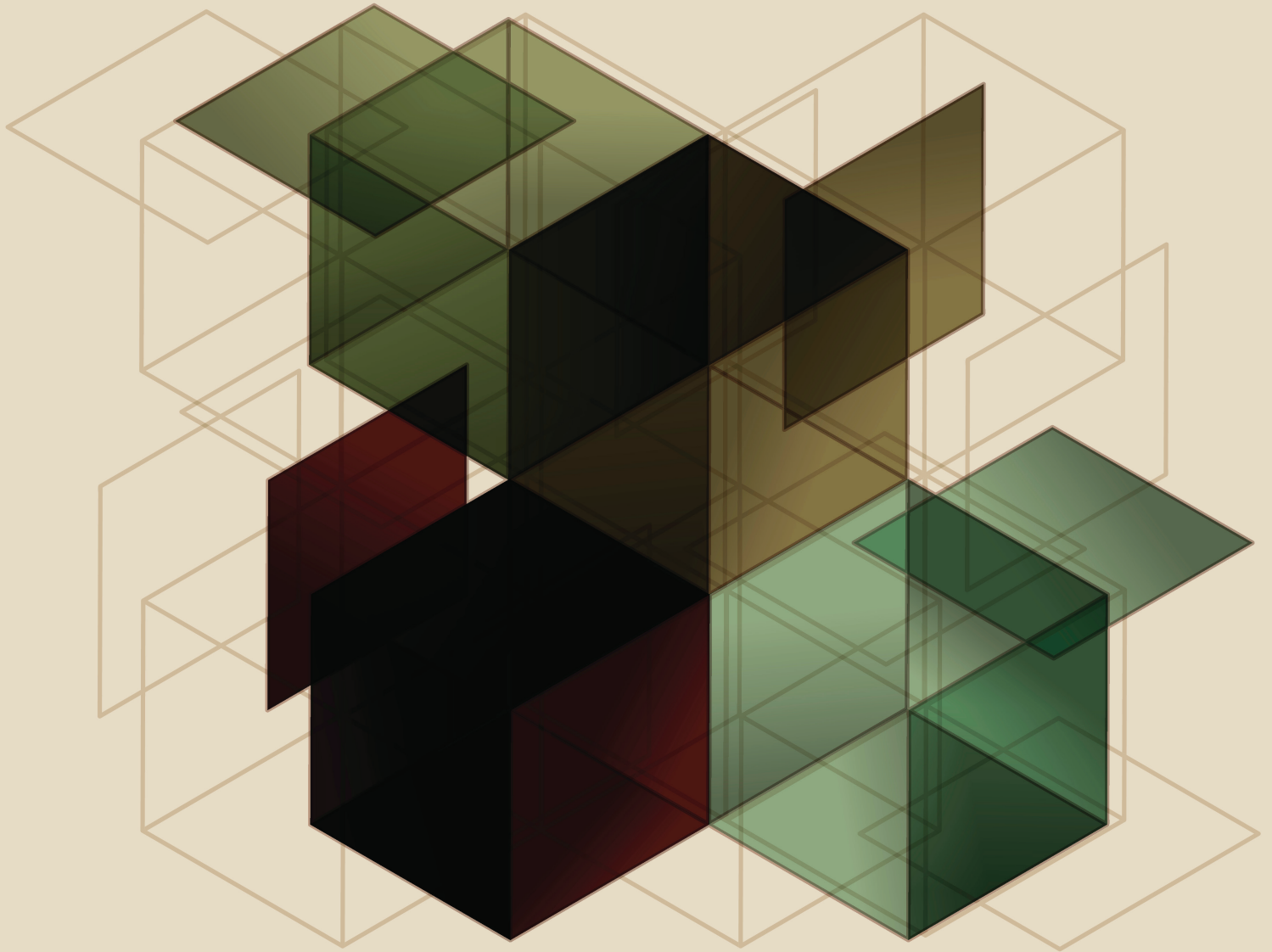


Upper Elementary Geometry



Geometry Task Cards Answer Key

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Answer Key

Nomenclature Review Lines:

Card 1: Answers will vary

Card 2:

- No parallel lines never meet
- Yes, if they are in the same plane
- Only if you know which direction the lines are going in. (arrows need to be marked on the segments to designate them as lines)

Card 3: Answers will vary

Card 4: Answers will vary

Nomenclature Review Angles

Card 1:

- No, largest acute angle is 89° ; four times 89 is less than 360°
- Yes, ie. $30 + 60 = 90$
- No, smallest reflex angle is 181° ; 2×181 is greater than 360
- Answers will vary

Card 2:

Answers will vary; guide to the puzzle is included.

Card 3:

- Yes, an acute angle is 60° so $3 \times 60 = 180$
- Yes, one obtuse equals 80° ; one acute is 40° which equals a reflex angle.
- Yes

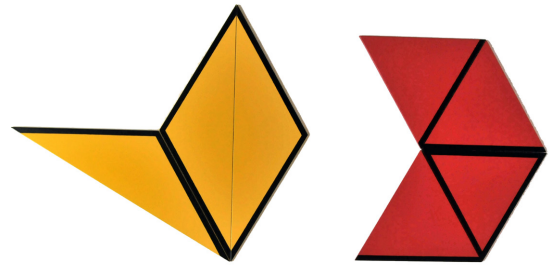
Card 4: Answers will vary

Triangle Box

Card 1:

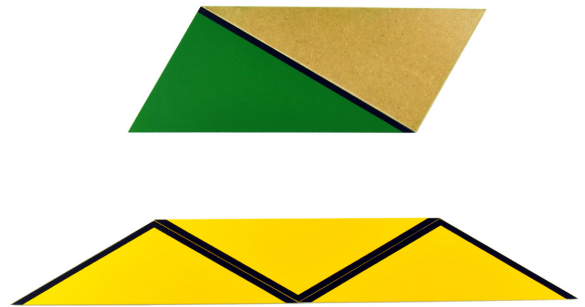
They are equivalent through the transitive property.

Construction:



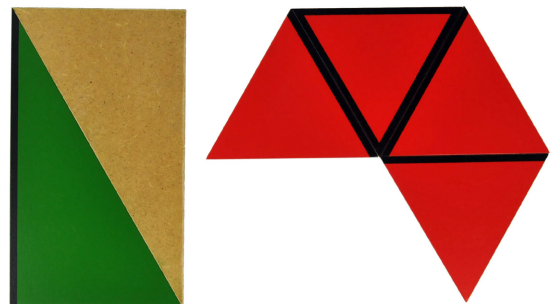
They are equivalent through the transitive property.

Construction:



They are equivalent through the transitive property.

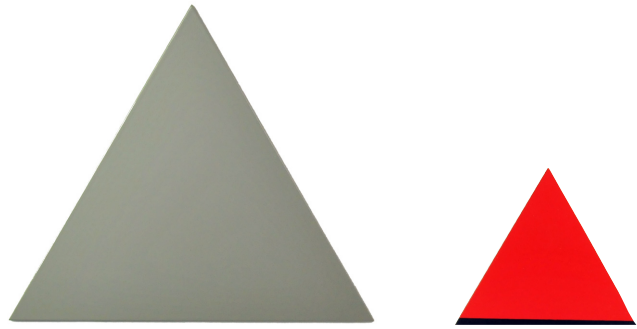
Construction:



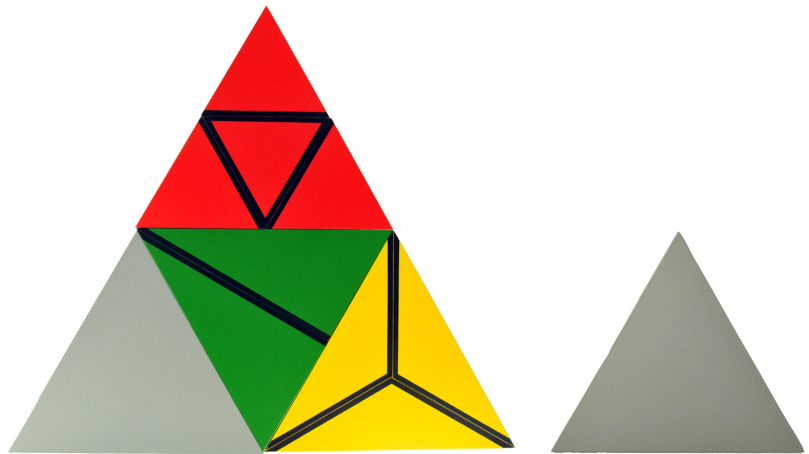
Card 2:

The large gray triangle and the small red triangle are similar figures with a 2:1 proportion.

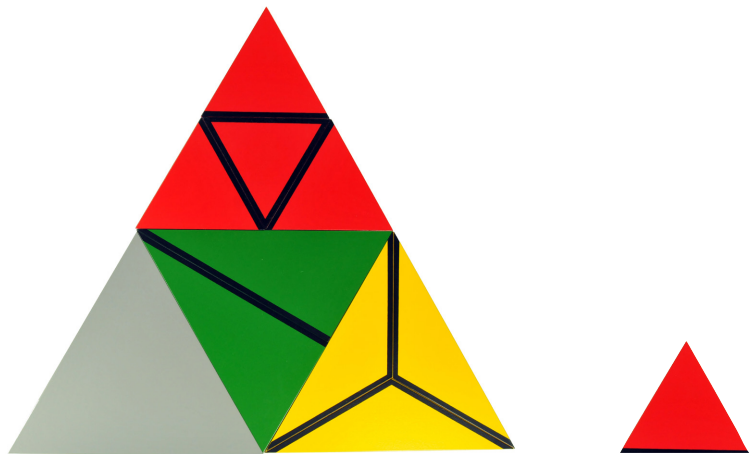
Construction:

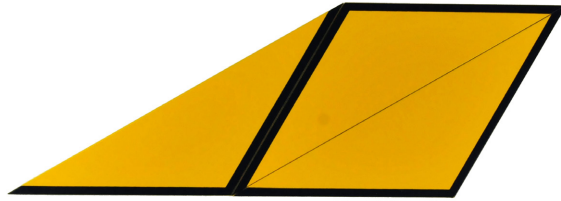


2:1 Proportion



4:1 Proportion





Obtuse Angle Trapezoid



Isosceles Angle Trapezoid

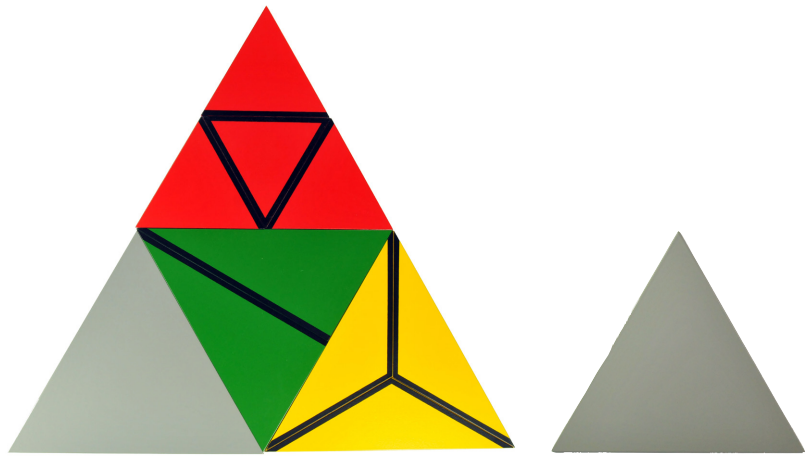
Both trapezoids are equivalent to each other.



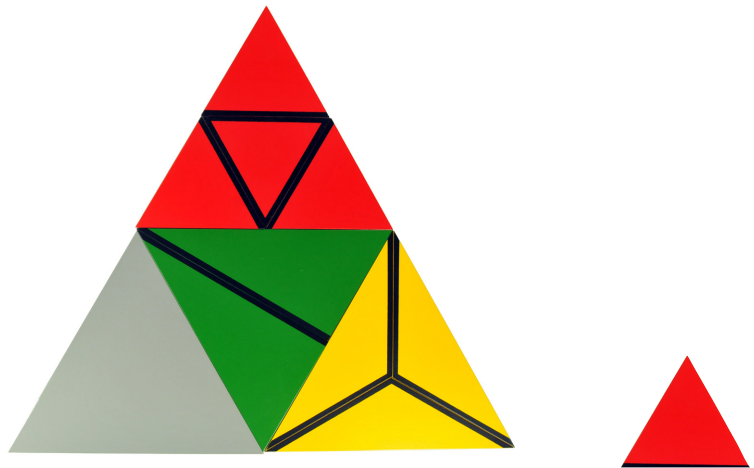
Right Angle Trapezoid

Card 3:

The fractional value of the guide triangle to the largest triangle is $\frac{1}{4}$



The fractional value of the small red triangle to the largest triangle is $\frac{1}{16}$



Proportionally the large parallelogram is 2:1 to the small parallelogram. Fractionally the small parallelogram is $\frac{1}{4}$ of the large parallelogram.



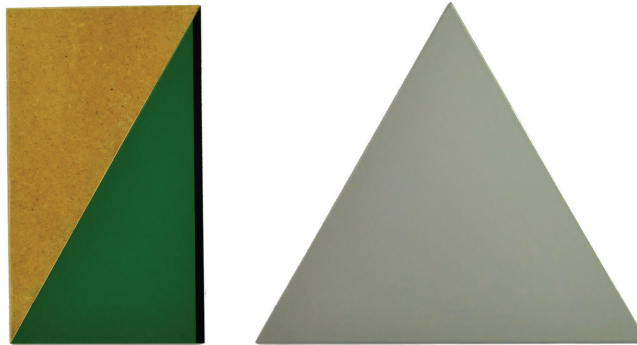
Card 3 (Continued):

This large trapezoid has a 2:1 proportion of the trapezoid built with the three red triangles.



Card 4:

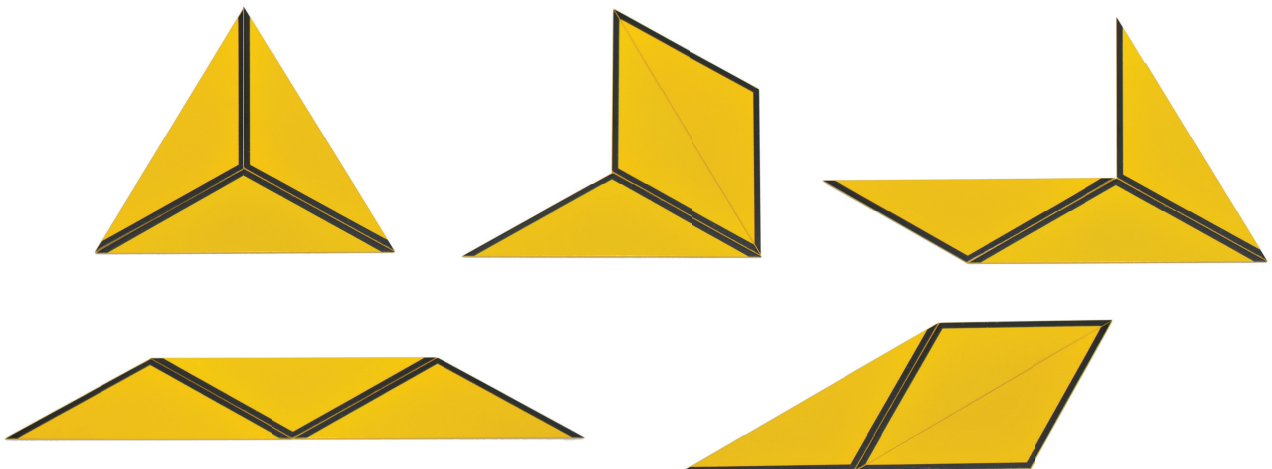
The height of the rectangle is equal to the height of the triangle; the base of the rectangle is $\frac{1}{2}$ of the triangle.



Large Hexagon Box

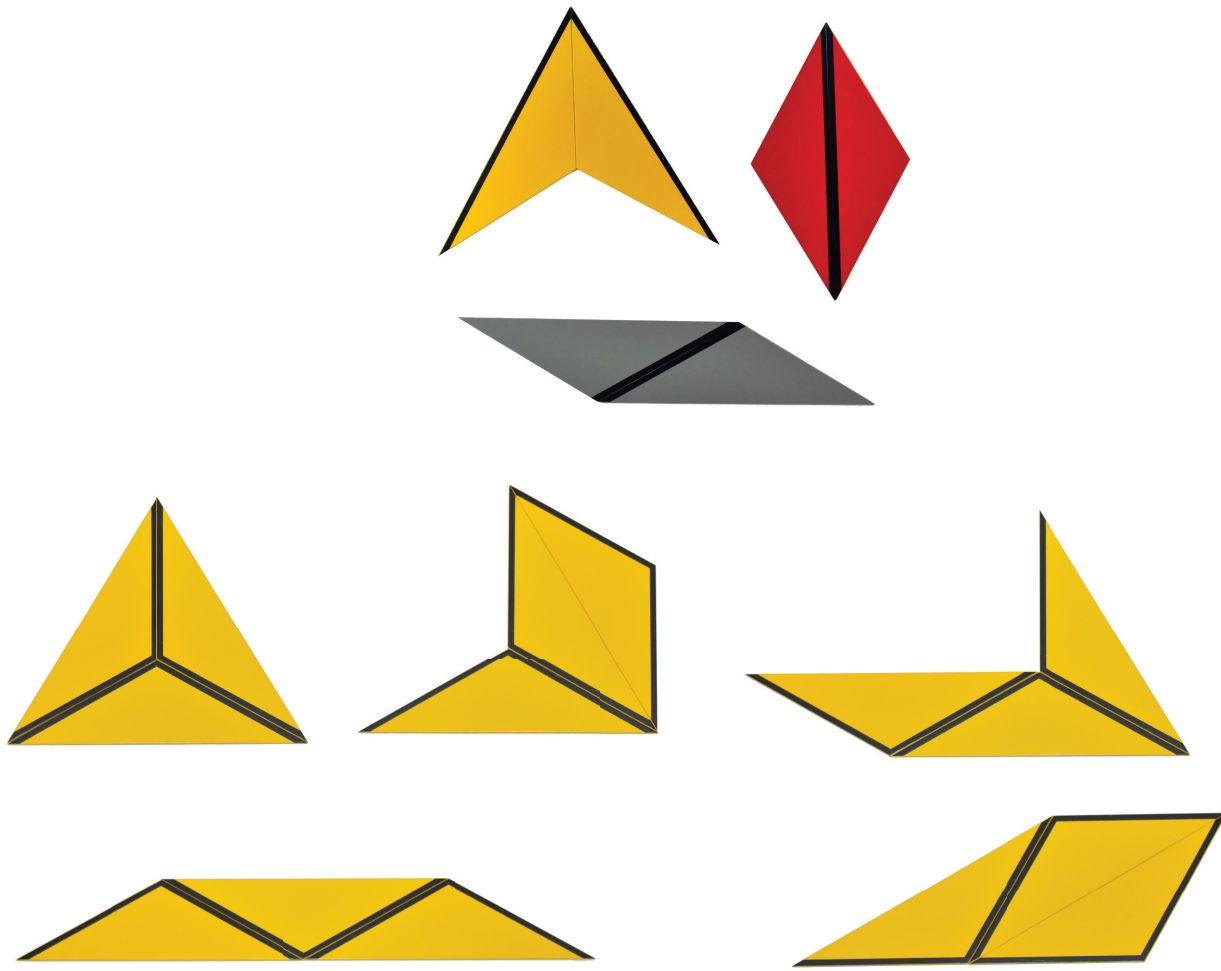
Card 1:

All of these figures are equivalent to the yellow equilateral triangle.



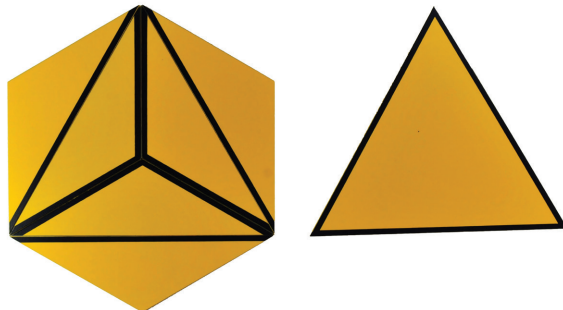
Card 2 (Continued):

Each of the figures made with the two isosceles obtuse triangles is $\frac{2}{3}$ of the figures made with the three isosceles obtuse triangles.



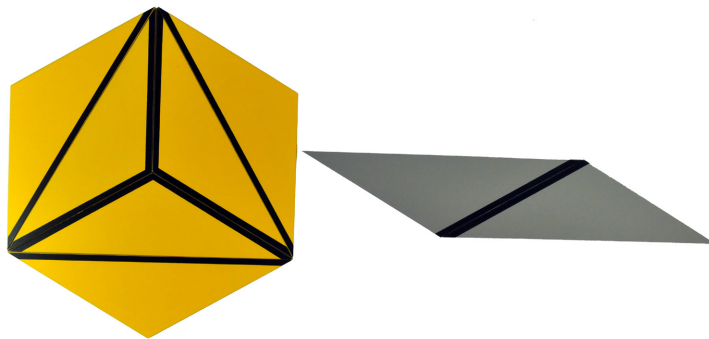
Card 3:

The yellow hexagon is two times the size of the yellow equilateral triangle.

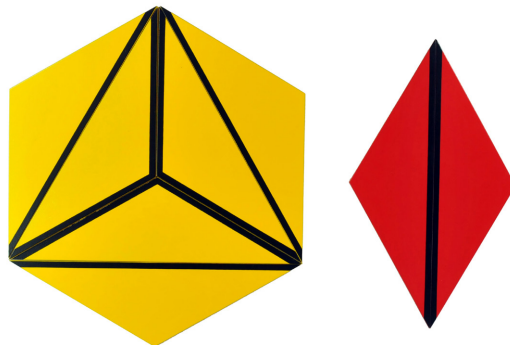


Card 3 (Continued):

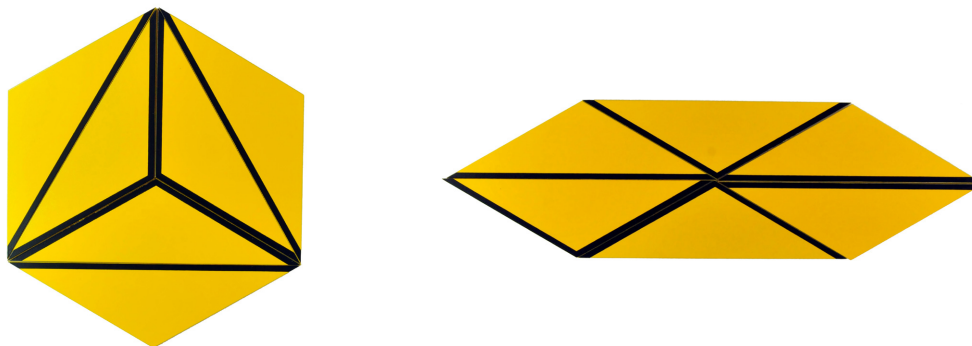
The parallelogram is $\frac{1}{3}$ of the yellow hexagon.



The rhombus is $\frac{1}{3}$ of the yellow hexagon.

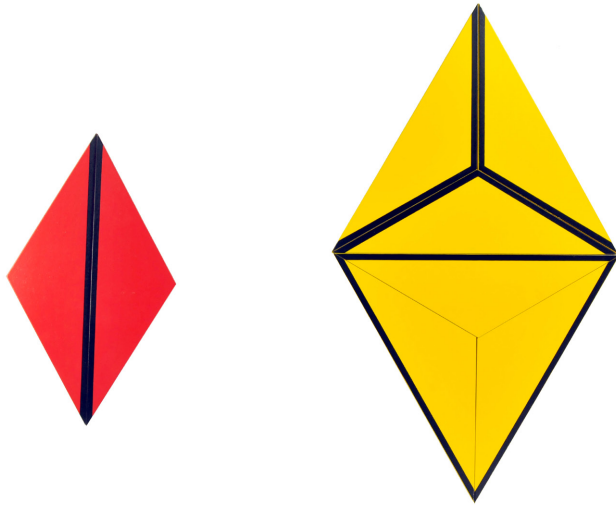


These two hexagons are equivalent.



Card 4:

The small red rhombus is $\frac{1}{3}$ of the largest rhombus.



The long diagonal of the red rhombus is equal to the length of the short diagonal of the yellow rhombus.

Card 5: Answers will vary

Card 6: Answers will vary

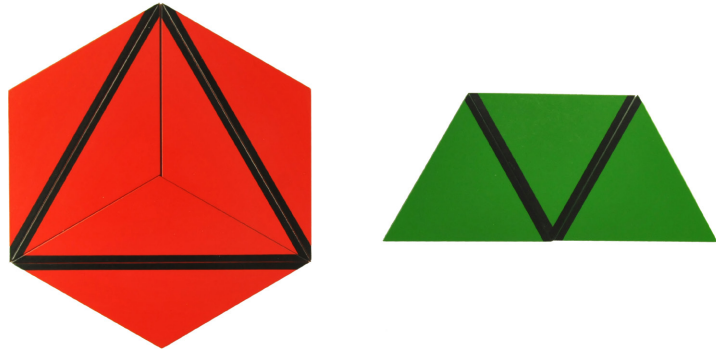
Card 7: Answers will vary

Card 8: Answers will vary

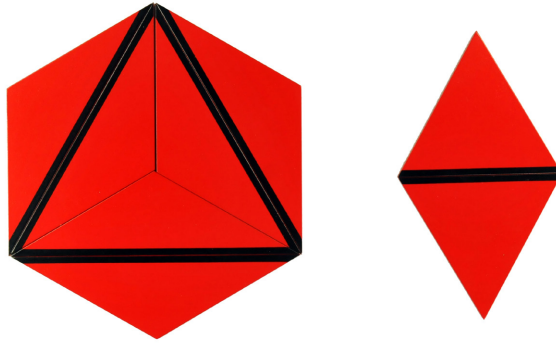
Small Hexagon Box (The key translator is the equivalency of the red equilateral triangle and the red isosceles obtuse triangle).

Card 1:

The trapezoid is $\frac{1}{2}$ of the hexagon. Reasons given will vary.



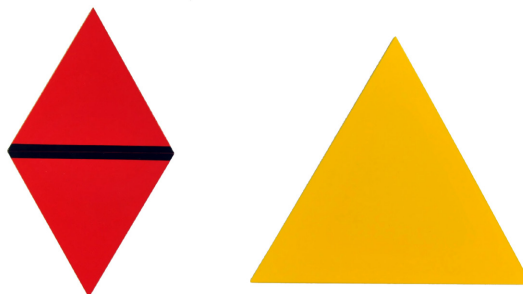
The red rhombus is $\frac{1}{3}$ of the red hexagon. Reasons given will vary.



The trapezoid is equivalent to the yellow equilateral triangle.

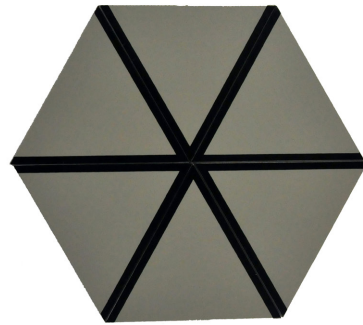
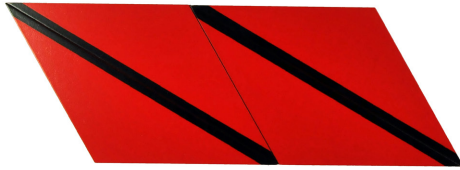


The rhombus is $\frac{2}{3}$ of the yellow equilateral triangle.

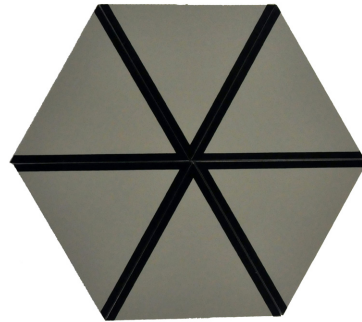
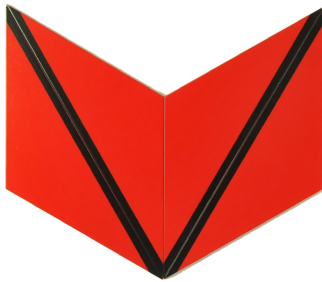


Card 2:

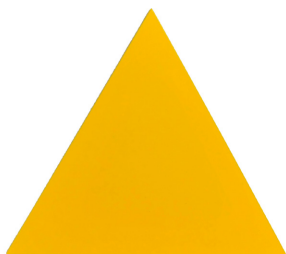
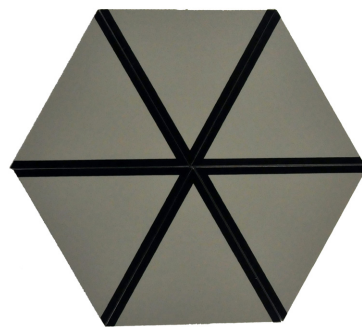
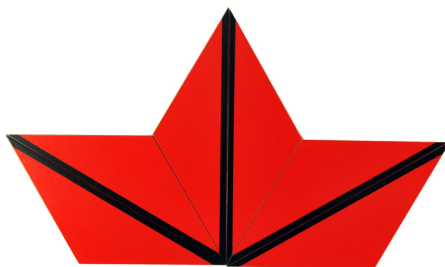
The parallelogram is $\frac{2}{3}$ of the hexagon. Reasons given will vary.



The concave hexagon is $\frac{2}{3}$ of the gray hexagon. Reasons given will vary.



The Fleur de Lis is equivalent to the gray hexagon. Reasons given will vary.



- The parallelogram is $1\frac{1}{3}$ of the equilateral triangle.
- The concave hexagon is $1\frac{1}{3}$ of the equilateral triangle.
- The Fleur de Lis is twice the size of the equilateral triangle.

Card 3: Answers will vary

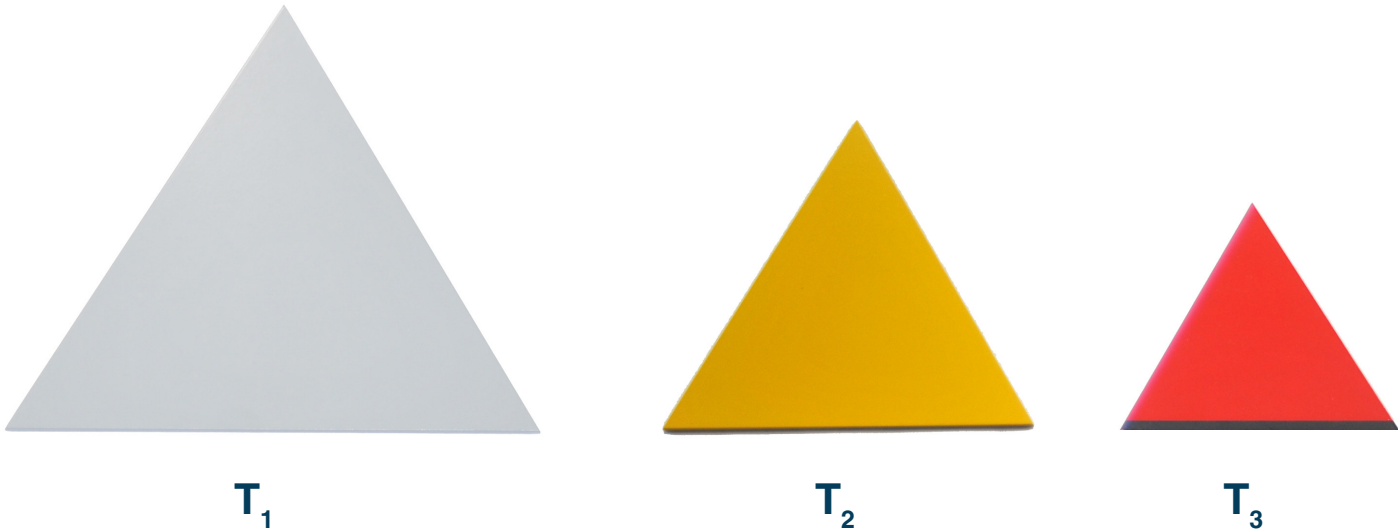
Card 4: Answers will vary

Card 5: Answers will vary

Card 6: Answers will vary

Comparison of Triangles (The key relationships are: T_2 is $\frac{3}{4}$ of T_1 , and T_3 is $\frac{1}{3}$ of T_2 , and T_3 is $\frac{1}{4}$ of T_1).

Card 1:



The scalene right triangle made from T_3 and the red isosceles obtuse triangle is equivalent to the green triangle in the triangle box.

This composite figure is $\frac{1}{2}$ of T_1 .

Card 2: Answers will vary

Card 3: Answers will vary

Card 4: Answers will vary

Card 5: Answers will vary

Card 6: Answers will vary

Card 7: Answers will vary

Card 8: Answers will vary

Note: The explorations of the relationships between the different triangles are to provide children with multiple opportunities to prove this mathematical relationship without using the sensorial aspects of the material.

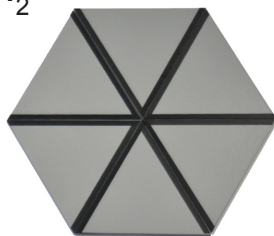
Pieces and labels necessary to do comparative hexagons are provided on the opposite page.

Note: Pieces and labels necessary to do comparative hexagons are provided on the opposite page.

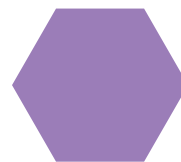
H_1



H_2



H_3



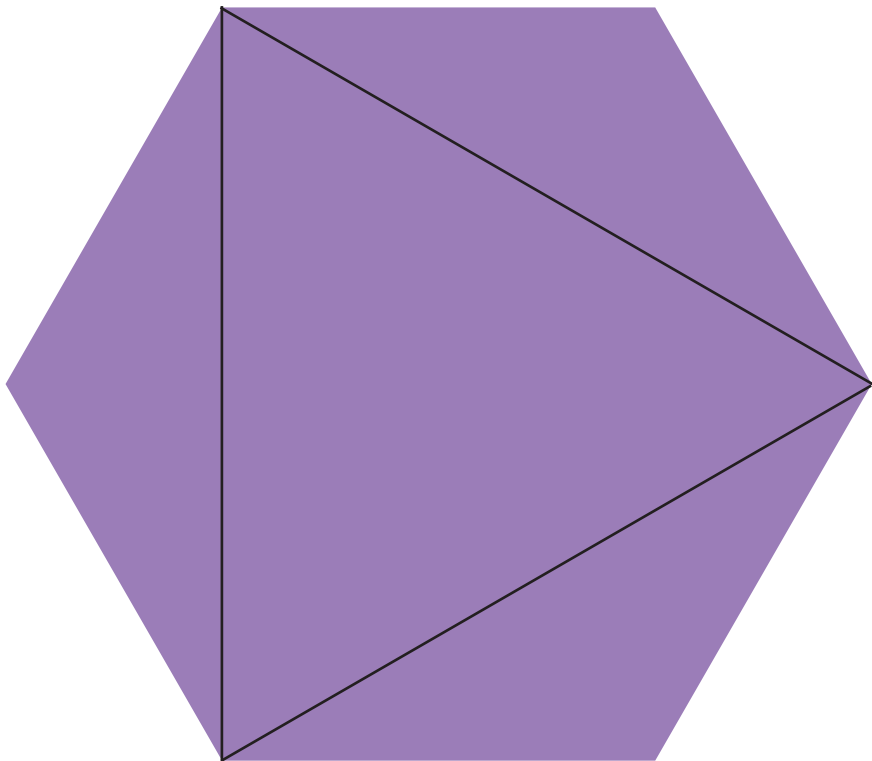
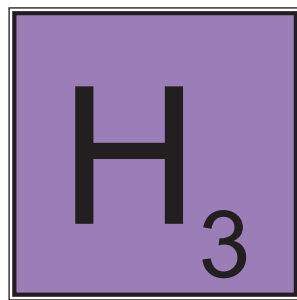
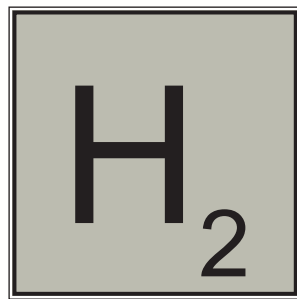
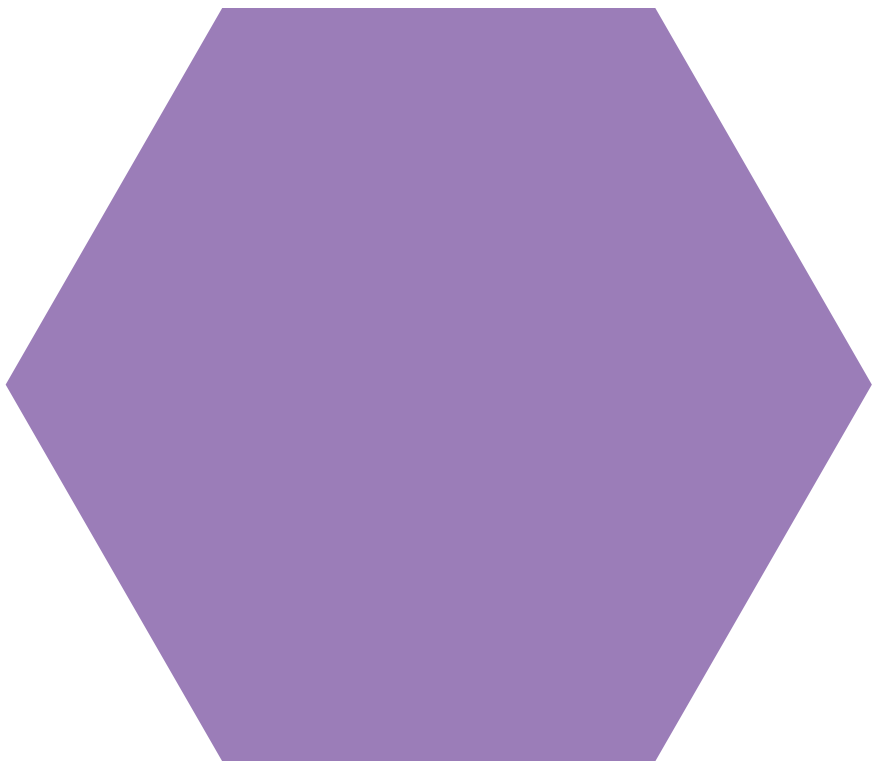
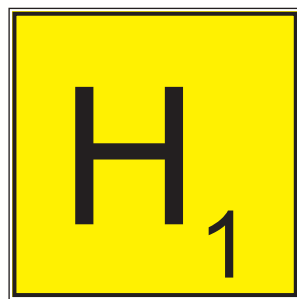
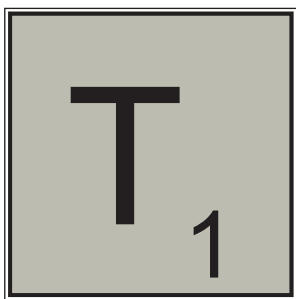
Comparison of Hexagons (The key relationships are: H_2 is $\frac{3}{4}$ of H_1 , and H_3 is $\frac{1}{3}$ of H_2 , and H_3 is $\frac{1}{4}$ of H_1).

Card 1: Answers will vary

Card 2: Answers will vary

Card 3: Answers will vary

Card 4: Answers will vary



Nomenclature Review of Triangles:

Card 1: The sum of all angles in any given triangle equals 180°

Card 2:

- No, any angle larger than an acute angle will cause the triangle to NOT be equilateral.
- No, shortening the hypotenuse does not give a right angled triangle.
- No, two obtuse angles will not make any triangle.

Card 3: Answers to the puzzles are included in the puzzle packet.

Card 4: Answers will vary

Equivalency Commands for Parallelograms

Card 1:

Sample Answer: Cut a vertical line along the altitude. The resulting figures are a right angle trapezoid and a scalene triangle. Translate the right scalene triangle so that the hypotenuses of each figure are touching, the resulting figure is a rectangle.

Card 2:

Sample Answer: Cut a vertical line along the altitude. The resulting figures are a right angle trapezoid and a scalene triangle. Translate the right scalene triangle so that the hypotenuses of each figure are touching, the resulting figure is a rectangle.

Card 3:

Sample Answer: Cut a vertical line along the altitude. The resulting figures are a right angle trapezoid and a scalene triangle. Translate the right scalene triangle so that the hypotenuses of each figure are touching, the resulting figure is a rectangle.

Card 4:

Sample Answer: When transforming the parallelogram into a rectangle the resulting rectangle will have the same height and base measurements.

Equivalency Commands Triangles:

Card 1:

Sample Answer: Cut a horizontal line from the mid-point of the hypotenuse to the midpoint of the opposite side. The resulting figure is a right trapezoid and a right triangle. Rotate the right triangle so that the hypotenuses of the two figures are touching. The resulting figure is a rectangle with a base equal to the base of the original triangle.

Card 2:

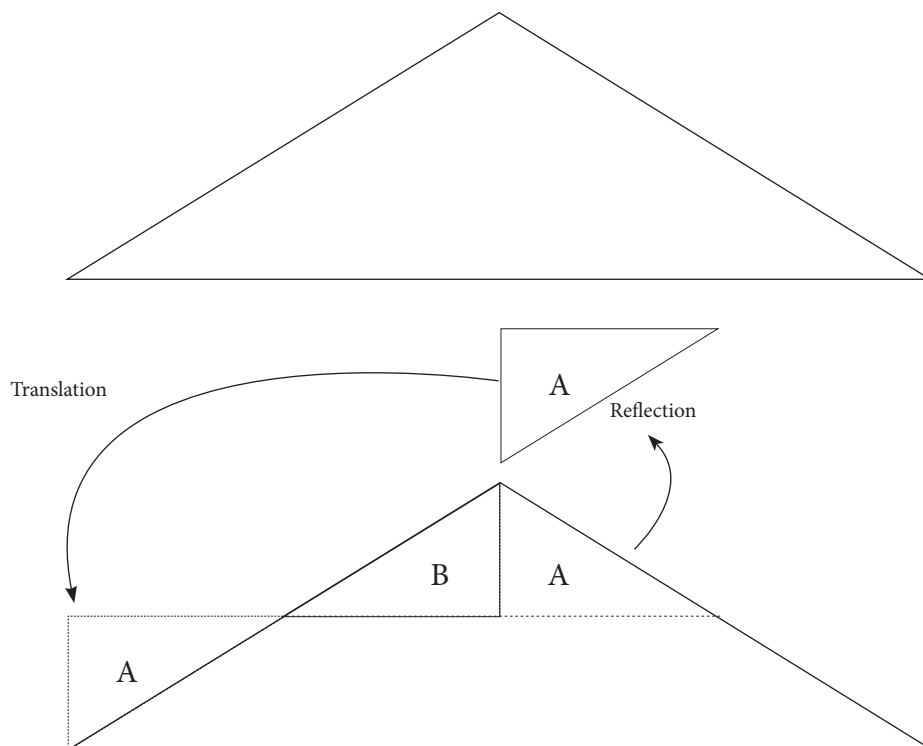
Sample Answer: Cut a horizontal line from the mid-point of one isosceles side to the mid-point of the opposite side. The resulting figures are an isosceles trapezoid and an isosceles acute triangle. Cut a vertical line on the altitude of the triangle. The resulting figures are two right scalene triangles. Rotate each right scalene triangle so that the hypotenuse touches each side of the isosceles trapezoid. The resulting figure is a rectangle with a base equal to the original triangle.

Card 3:

Sample Answer: Cut a horizontal line from the mid-point of the hypotenuse to the midpoint of the opposite side. The resulting figure is a right trapezoid and a right triangle. Rotate the right triangle so that the hypotenuses of the two figures are touching. The resulting figure is a rectangle with a base equal to the base of the original triangle.

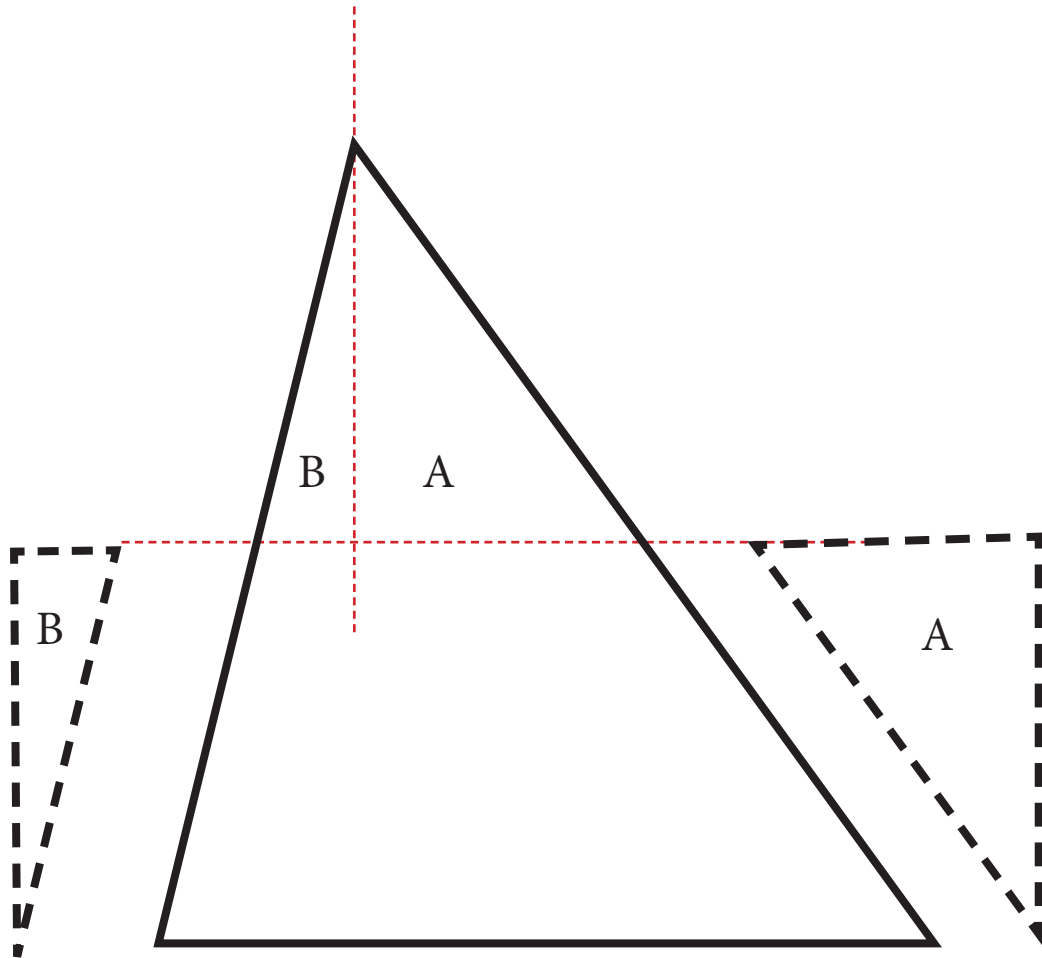
Card 4:

Sample Answer: Orient the triangle so that the base is the longest side. Cut a horizontal line from the mid-point of each isosceles side. The resulting figures are an isosceles trapezoid and a smaller obtuse triangle. Cut the smaller obtuse triangle on its altitude to get two right triangles (A, B). Reflect each right triangle and translate to the opposite isosceles sides of the trapezoid.



Card 5:

Sample Answer: Cut a horizontal line from the mid-point of one side to the mid-point of the opposite side. The resulting figures are an trapezoid and an acute scalene triangle. Cut a vertical line on the altitude of the triangle. The resulting figures are two right scalene triangles. Rotate each right scalene triangle so that the hypotenuse touches each side of the isosceles trapezoid. The resulting figure is a rectangle with a base equal to the original triangle.



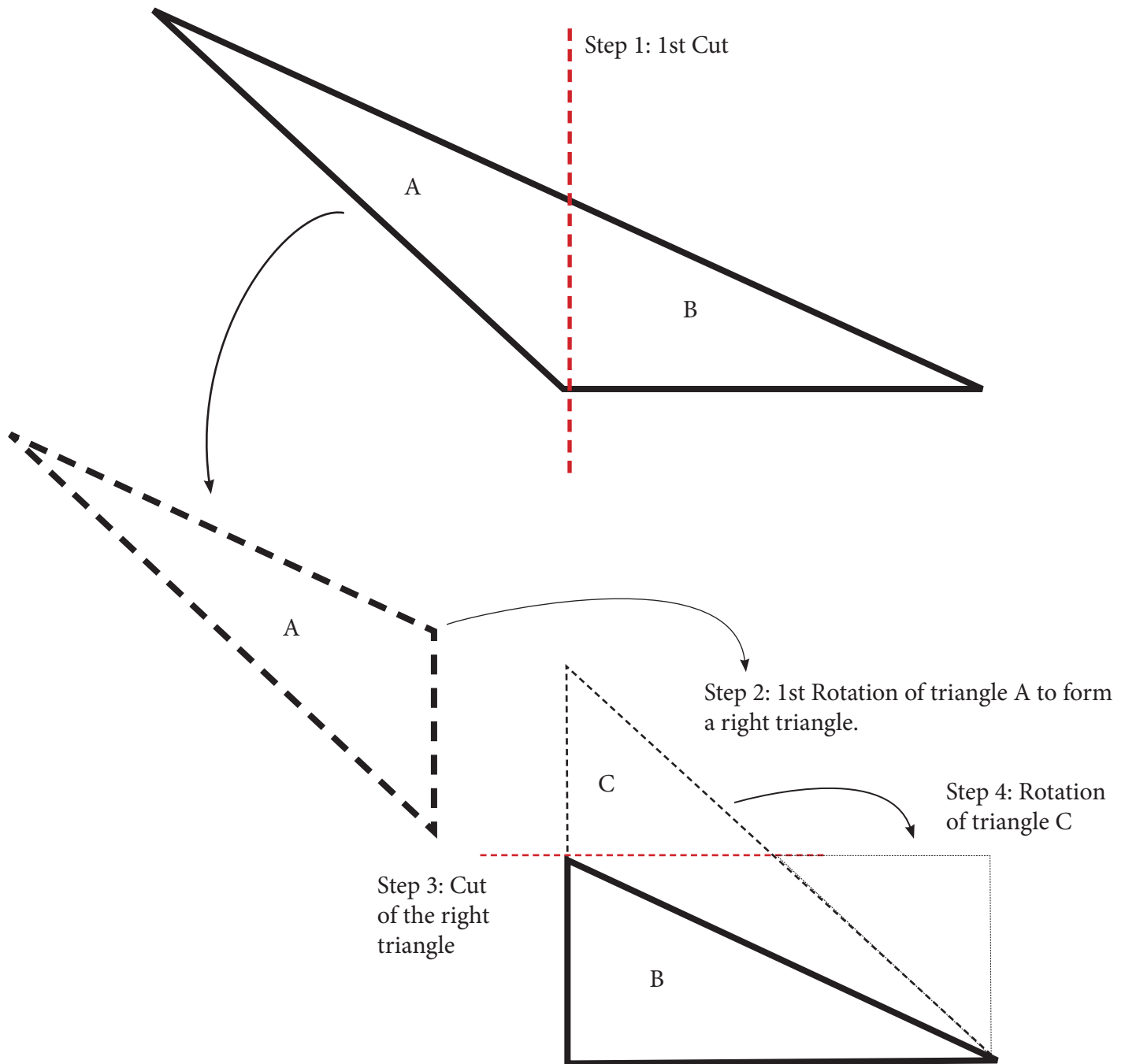
Card 6:

Sample Answer: Starting with a right triangle, make the hypotenuse your base. Cut along the altitude; the resulting figures are two right scalene triangles. Rotate one triangle so that it touches the hypotenuse of the second triangle. The resulting figure is a square which is a special rectangle.

Card 7:

Sample Answer: Cut from the midpoint of the longest side to the opposite vertex. The resulting figures are a right scalene triangle (B) and an obtuse scalene triangle (A). Rotate the obtuse scalene triangle (A) to touch the hypotenuse of the right triangle (B). The new figure is a right isosceles triangle.

Make a perpendicular cut from the midpoint of an isosceles side to the hypotenuse. The resulting figures are a right trapezoid and a right triangle (C). Rotate the right triangle (C) so that the hypotenuses touch. The resulting figure is a rectangle.



Card 8:

Sample Answer: Resulting rectangles formed from triangles either have half the base and the full height of the triangle, or half the height and the full base of the triangle.

Nomenclature Review for Quadrilaterals.

Card 1:

Sample Answer: Six quadrilaterals from least perfect to most perfect are:

1. Common Quadrilateral
2. Trapezoid
3. Parallelogram
4. Rhombus
5. Rectangle
6. Square

Question 1: Does it have four sides?

Question 2: Does it have one set of parallel sides?

Question 3: Does it have two sets of parallel sides?

Question 4: Are the parallel sides of equal length?

Question 5: Does it have four right angles?

Question 6: Does it have all of the above?

Card 2: Answers will vary

Card 3: Answers will vary

Card 4: Answers will vary

Equivalency Commands for Rhombus.

Card 1:

Sample Answer: Cut along the short diagonal. The resulting figures are two equilateral triangles. Cut along the altitude of one of the equilateral triangles resulting in two right scalene triangles. Rotate each right scalene triangle to touch the side of the equilateral triangle. The resulting figure is a rectangle.

Card 2: Answers will vary

Card 3:

Sample Answer: Cut along the long diagonal. The resulting figures are two isosceles obtuse triangles. Cut along altitude of one of the isosceles obtuse triangle. The resulting figures are two right scalene triangles. Rotate each right scalene triangle to touch the isosceles sides of the isosceles obtuse triangle. The resulting figure is a rectangle.

Card 4: Answers will vary

Card 5: Answers will vary

Card 6:

Sample Answer: The rectangle that results from a rhombus has the measurements of either half of the short diagonal and the full length of the long diagonal, or half of the long diagonal and the full length of the short diagonal.

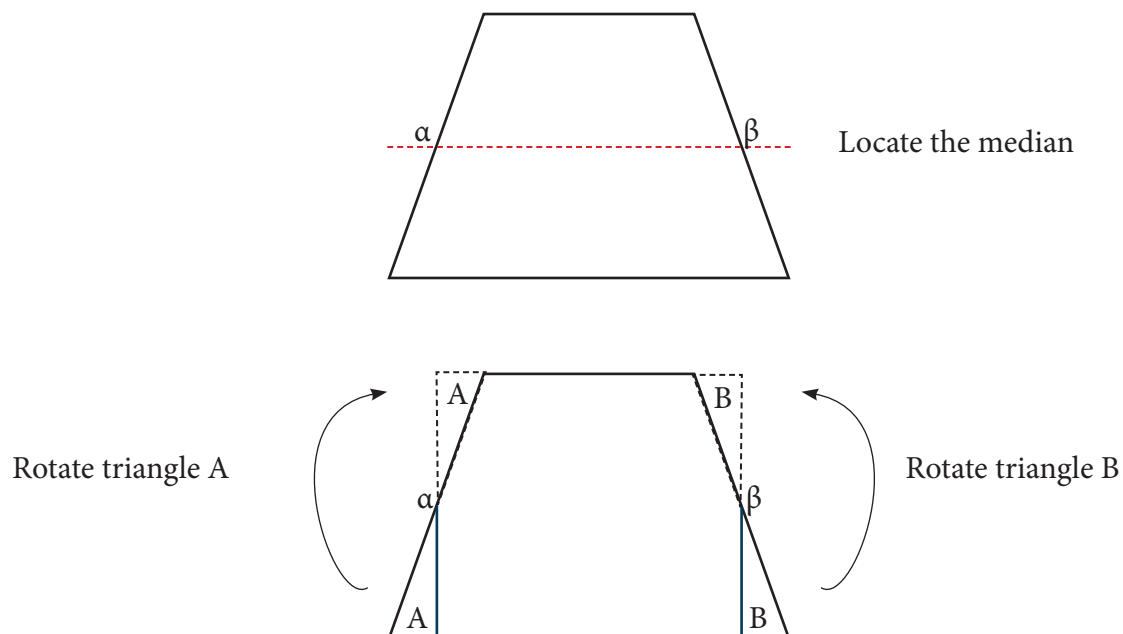
Equivalency Commands for Trapezoids.

Card 1:

Sample Answer: Cut a horizontal line from the midpoint of one side to the midpoint of a hypotenuse (median). The resulting figures are two right trapezoids. Rotate the smaller right trapezoid so that the hypotenuses touch. The resulting rectangle has a base of 20 cm.

Card 2:

Sample Answer:



Locate the median of the trapezoid. Construct a horizontal line along the median points (α , β). Cut a perpendicular line from α point to the major base of the trapezoid. Repeat the step but this time do it from point β . The resulting figure is a rectangle and two right scalene triangles. Rotate triangle A 180°. Repeat the step for triangle B. The resulting figure is a rectangle.

Card 3: Answers will vary

Card 4:

Sample Answer: The height of the rectangle from the trapezoid will either be the full height of the trapezoid and half of the measurement of the major base and minor base, or, the full measurement of the major base and minor base and half of the height.

Nomenclature Review for Polygons.

Card 1: Answers will vary

Card 2:

Sample Answer: No the intersection of the diagonals in a polygon are not always in the center.

The radius of a polygon is equal to half of the diagonal.

Yes, they are the same number of radii as apothems in a polygon. A radius goes from the midpoint of the polygon to the vertex, an apothem goes from the midpoint of the polygon to the midpoint of the side.

Card 3:

Sample Answer:

Calculating the Number of Diagonals in a Polygon

As the children are exploring the number of diagonals in each polygon, a pattern will appear. Although not expected, there are children who are able to translate this pattern into a formula that can be used for any polygon.

	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon	Nonagon	Decagon
Sides	3	4	5	6	7	8	9	10
Vertices	3	4	5	6	7	8	9	10
Diagonals	0	2	5	9	14	20	27	35
Radii	3	4	5	6	7	8	9	10
Apothems	3	4	5	6	7	8	9	10

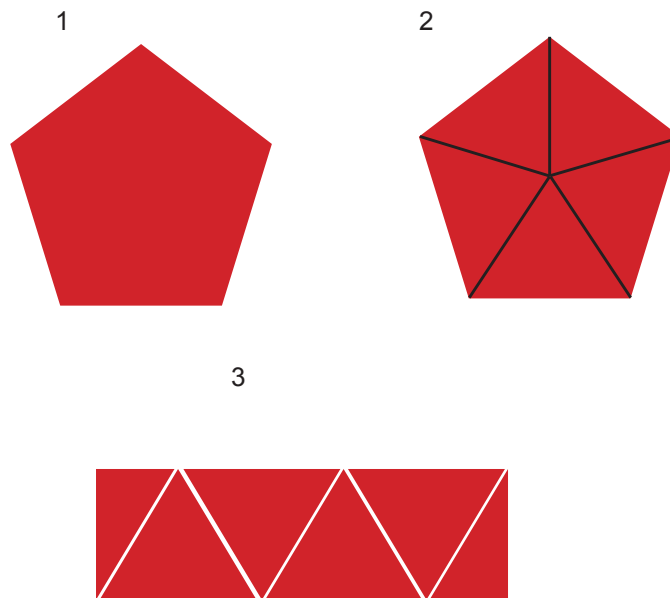
The formula for calculating the number of diagonals in any given polygon is:

$$\frac{(x-3)x}{2} \quad \text{where } x = \# \text{ of vertices.}$$

Card 4: Answers will vary

Equivalency Commands for Polygons.

Card 1:



1. Cut five isosceles acute triangles by drawing a line from the center of the pentagon to each vertex.
2. Cut one of the isosceles acute triangles along the altitude to form two scalene right triangles.
3. Place the isosceles triangles so they form a parallelogram and place the hypotenuse of the scalene right triangles along the oblique line of the parallelogram.
4. A rectangle will be formed.

Card 2: Same as card 1

Card 3: Same as card 1

Card 4: Same as card 1

Card 5: Same as card 1

Card 6: Same as card 1

Card 7: Same as card 1

Card 8:

Sample Answer: The rectangle formed by cutting a polygon has as its base half of the perimeter and the height of the apothem.

Nomenclature Review Circles.

Card 1:

Answers to crossword puzzle are given.

Card 2:

No, if you extend the chord of a circle it will not become tangent. A tangent line touches the circle at only one point. An extended chord touches the circumference at 2 points.

Yes, the diameters of a given circle are always the same measurement in length as a diameter must go through the center of the circle.

Card 3:

In the sixth fractional inset there are 6 arcs, 6 radii, and 3 diameters.

Answers will vary.

Card 4: Answers will vary.

Polyhedra

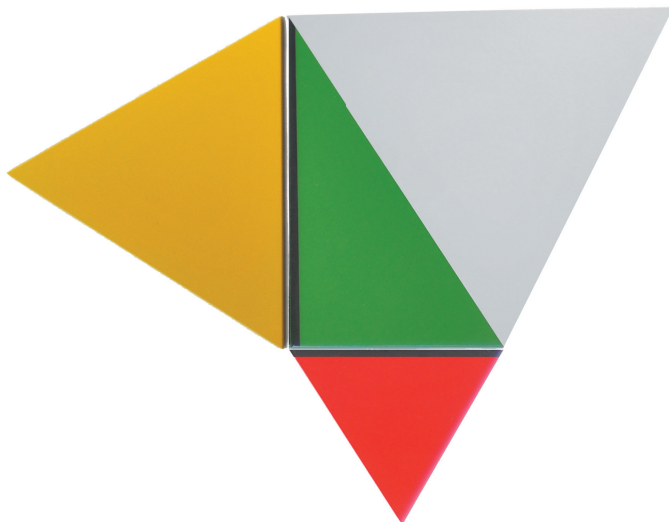
Cards 1-8:

Answers to these cards will vary and are based on what type of materials you are using. Therefore, no answers are provided.

Pythagoras

Card 1: Answers will vary depending upon the size of the triangles provided by your vendor. This work

will ask the students to use the relationship between T1, T2, and T3 to build composite figures and to use their area formulas to prove the Pythagorean theorem.



Card 3-4:

Answers will vary depending on what the length is of your materials and what vendor you have used to acquire your materials.

Card 5:

The length of your stabilizing piece of lumber will be 9.44 ft.

Card 6:

The length of the side is 1.32 m.

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