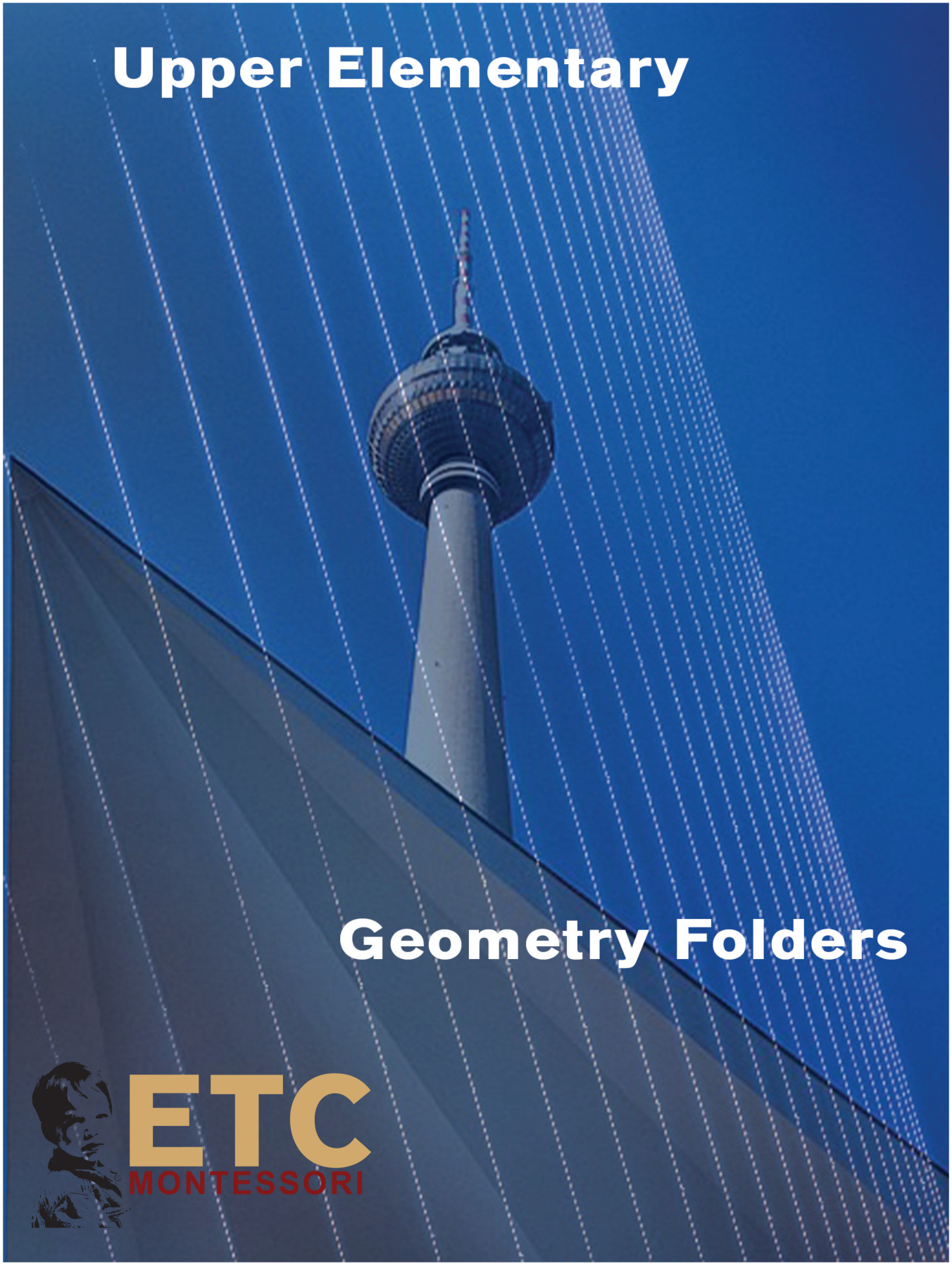
Upper Elementary



Geometry Folders

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Introduction to Area

Direct Aim: Development of understanding that area must always be measured in two dimensions. (squares)

Indirect Aim: Preparation for the development of the formulas of area of plane figures.

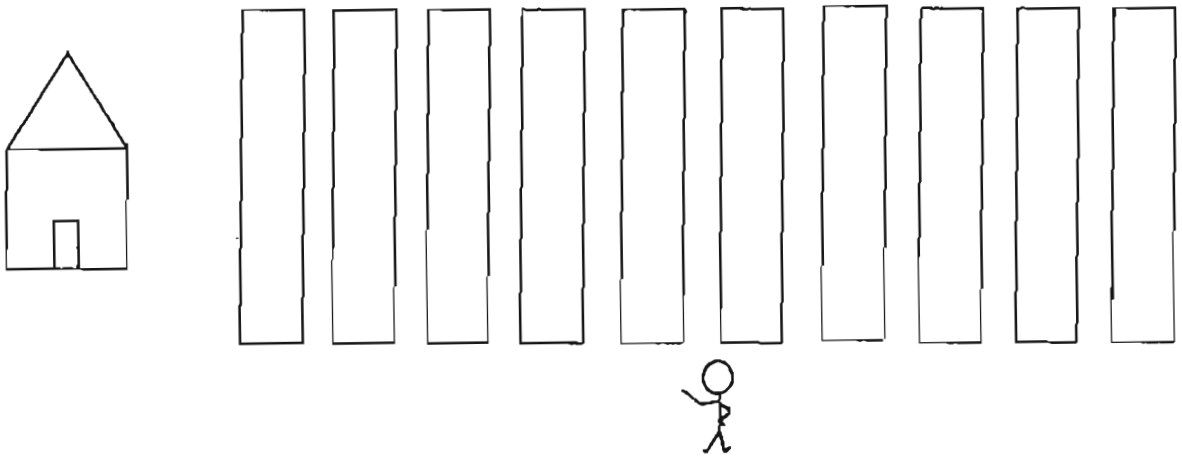
Prerequisites: Multiplication

Materials: • Yellow material for the study of area

Presentation:

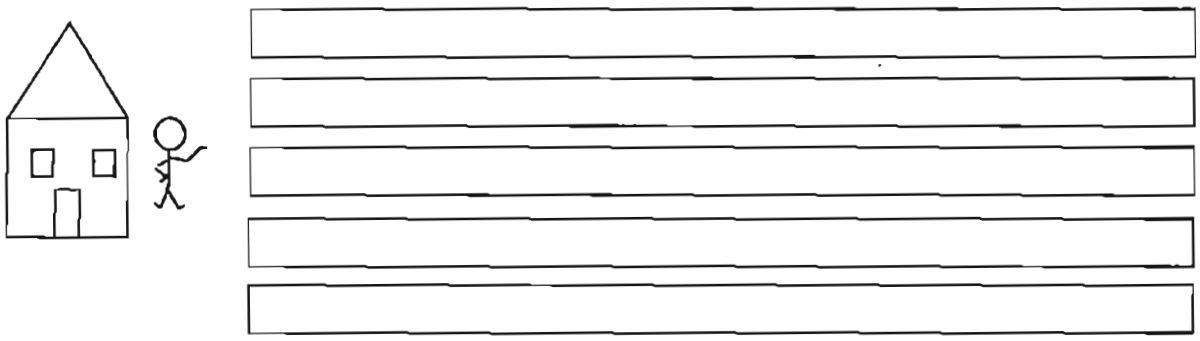
1. This work is introduced with a story. We will begin with the area of a rectangle. Each subsequent figure will also be presented with a story.

Story: Long, long ago people gathered the grains and seeds that they saw grew around them. These were brought into their homes. Some grains spilled and were eaten by animals. Some took root and grew where they fell. People thought that they owned the grain that grew near their houses. Some gathered the grain and grew it in random places. Soon there were fields of grain. One family lived in a house surrounded by grain. There were two sons and a young daughter.



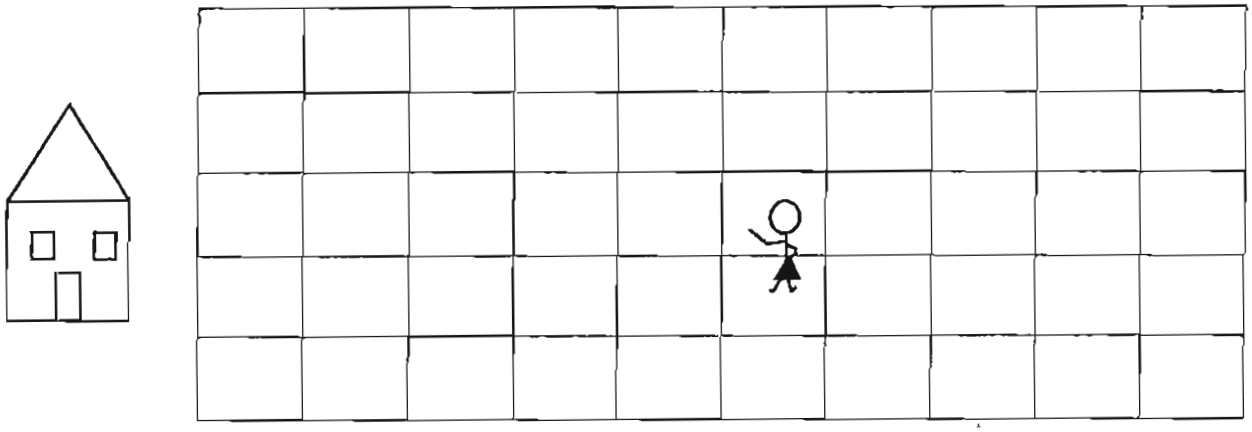
The girl's name was Marigold. The old farmer was ready to give his fields to his sons, but he didn't know how big it was. He offered the land to his sons, if they could tell him how big it was. The oldest son went out to the field and pondered the question. He noticed that the wheat grew in ten rows.

(Take out the rectangle with ten rows along the base.) He came back to his father and informed him that the field was 10 big. The father was so proud of his son that he threw a party and invited the whole town to come and share in the handing over of the inheritance. While everyone was preparing for the party, the second son went out into the field. He examined the field and found that he disagreed with his older brother.



He saw that there were only five rows. (Place the yellow rectangle with five rows along its height on top of the previous yellow rectangle.) He went to call his father to show him that his older brother was wrong. The father came out and agreed with his son. So party preparations continued, only the heir changed. Then young Marigold went out into the wheat field. and she noticed:

Looking in one direction she saw five rows, but looking in the other she saw ten. So she counted the little squares, and realized that the property had 50 squares. (Place the yellow rectangle divided into squares on top.)



2. When the story is complete, the final piece, a rectangle with the area squares clearly marked, should be on the rug. Ask the student what is the area of the rectangle.

3. Use slips of paper to mark the base and the height of the rectangle. Write the formula: $A = b \cdot h$. (Explain that it is difficult to understand the formula if we use an "x" for times, so we drop the "x" and just place a •, or use nothing.)

4. Have the students write in their notebook the formula for the area of a rectangle and then replace the letters with measurements of areas of rectangles in the classroom.

$A = bh$

$A = 11\text{in} \bullet 8\text{ in}$

$A = 88$ square inches (cover of math book)

5. Students should always be required to write the formula whenever one is needed, then to exchange the variables for the knowns, then solve the problem.

Follow up: Word problems for area, for example Creative Publications Jobcards, or HMC geometry word problems. Students can measure any rectangles they find in their environment.

Area of the Parallelogram

Direct Aim: Development of a formula for the area of a parallelogram

Indirect Aim: Move into abstraction that which was done concretely with the equivalency of the parallelogram.

Prerequisites: Area of a rectangle

Materials: • Yellow material for the study of area

Presentation:

1. Hand the parallelogram to the student. Ask if she can count the squares. Acknowledge that it can't be done easily. We need a mediator.

2. Hand the student the divided parallelogram. Ask her to confirm if the two are congruent. Acknowledge that they are congruent. Set aside the one piece parallelogram.

3. Move the small triangle from one side of the parallelogram to the other. Ask the student to name this new figure. Rectangle. Ask if she remembers how to find the area of a rectangle.

4. Examine the rectangle and discuss why the parallelogram became a rectangle. A cut from the vertex to the base of the parallelogram formed a rectangle whose hypotenuse was exactly parallel to the other side of the parallelogram.

5. Develop a rule: A parallelogram is equivalent to the rectangle which has the same base and height. This can be written:

$$A = bh$$

Follow up: Geoboard, Creative Publications task cards, HMC geometry word problems, or other teacher made task cards.

Area of a Square

Direct Aim: Acknowledgement that a square is a rectangle with equal sides.

Indirect Aim: Development of formulae.

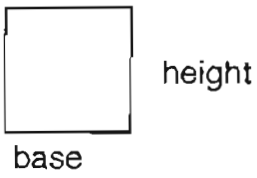
Prerequisites: Area of a rectangle.

Materials:

- Geometry notebook

Presentation:

1. Examine and describe the square. Find the base and the height.



2. Agree that a square's base and height are the same. Agree to call the base and the height "side".

3. Rewrite the area formula for a square:

$$A = s^2$$

Follow up: Geoboard, Creative Publications task cards, HMC geometry word problems, or other teacher made task cards.

Area of a Triangle

Direct Aim: Development of a formula for the area of a triangle.

Indirect Aim: Beginning of understanding of the commutative aspects of working with a formula.

Prerequisites: Area of the rectangle and parallelogram

Materials: • Yellow material for the study of area

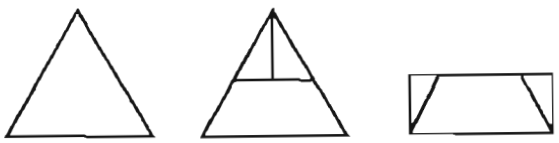
Presentation: Equilateral Triangle

1. Lay out all the triangles from the box of yellow material. Ask the student to name each triangle. Separate the equilateral triangle. Examine it carefully with the student and ask if he thinks he could possibly tell you the area. Ask why not? Agree that it is difficult because of all the half squares and quarter squares apparent in the triangle.

2. First Formula: Supply student with the equilateral triangle divided along the height. Ask the student if using these pieces he can find the area of the triangle.

3. Be certain to label the base and the height of the triangle before manipulating the pieces. Once the triangle is a rectangle, examine it carefully, finding the base and the height to discover the formula. Write the formula:

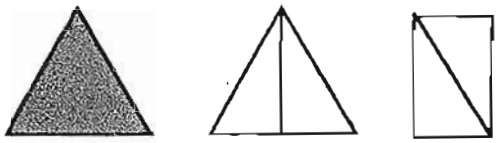
$$A = \frac{bh}{2}$$



4. Second Formula : Now give the student the equilateral triangle divided by the base. Label the base and the height. Ask where this triangle is divided. Ask the student if he can find the area of this equilateral triangle.

4. Build the rectangle using the divided pieces. Examine the rectangle to find the base and the height of the triangle. Label the parts. Notice that one half the base of the triangle is equal to the base of the rectangle.

6. Write the new formula:



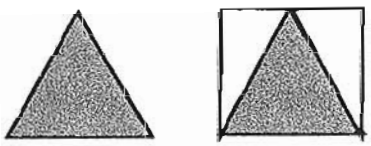
$$A = \frac{bh}{2}$$

7. Third Formula: Place the whole equilateral triangle on the rug. Once again ask the student to count the squares. Since this is difficult we will use the divided equilateral triangle as a mediator.

8. First, we must agree that the divided triangle is congruent to the whole triangle. Now we can flip the divided triangle up and see what we get. We have made a square. It is two equilateral triangle.

9. We can propose a new formula which states:

$$A = \frac{bh}{2}$$



10. This new formula is the general formula for the area of a triangle. The student must write all three formulas in their notebook.

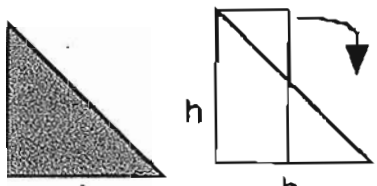
Presentation: Right Angled Triangle

1. Separate the right angled isosceles triangle from the box of yellow material. Examine it carefully with the student and ask if he thinks he could possibly tell you the area. Ask why not? Agree that it

is difficult because of all the half squares and quarter squares apparent in the triangle.

2. First Formula: Supply student with the right angled triangle that is divided . Ask the student if using these pieces are congruent to the whole right angled triangle.

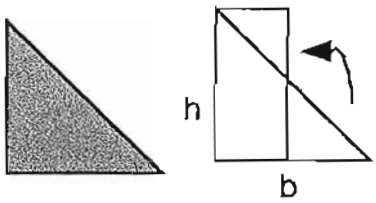
3. Be certain to label the base and the height of the triangle before manipulating the pieces. Once the triangle is a rectangle, examine it carefully, finding the base and the height to discover the formula. Write the formula:



$A = \frac{bh}{2}$

4. Second Formula : Now give the student the right angled triangle divided by the base. Label the base and the height. Ask where this triangle is divided. Ask the student if he can find the area of this right angled triangle.

5. Build the rectangle using the divided pieces. Examine the rectangle to find the base and the height of the triangle. Label the parts. Notice that one half the base of the triangle is equal to the base of the rectangle.

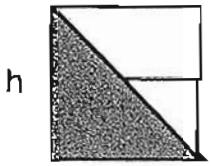


6. Write the new formula: $A = \frac{bh}{2}$

7. Third Formula: Place the whole right angled triangle on the rug. Once again ask the student to count the squares. Since this is difficult we will use the divided right angled triangle as a mediator.

8. First, we must agree that the divided triangle is congruent to the whole triangle. Now we can flip the divided triangle up and see what we get. We have made a square. It is two right angled triangle.

9. We can propose a new formula which states:



$$A = \frac{bh}{2}$$

10. This new formula is the general formula for the area of a triangle.

11. Student writes all three formulas in his notebook.

Follow up: Creative Publications task cards, HMC or teacher made task cards.

Area of a Rhombus

Direct Aim: Development of a formula for the area of a rhombus.

Indirect Aim: Application for real life area problems.

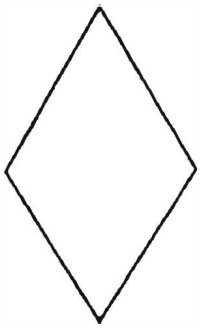
Prerequisites: Area of the rectangle.

Materials:
- Package of rhombus area material
- Geometry notebook

Presentation:

1. Using the small yellow pieces from the area package construct a rhombus. Ask the student to transform it into a rectangle.

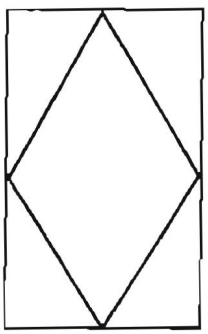
2. Discuss: Since we know the area of a rectangle and we know the area of a parallelogram, we should be able to say that the area of a rhombus is base times height.



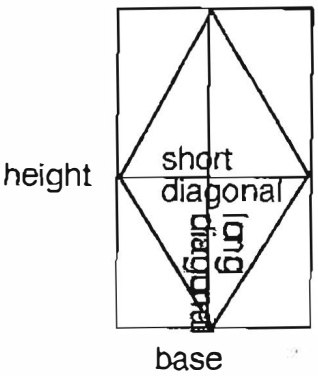
BUT

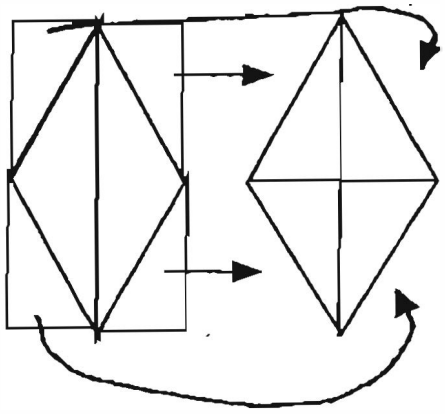
Where is the base? Where is the height?

3. Show the large pink rhombus (not marked). Where is the base? Where is the height? Show the rectangle. Where is the base? Height?



4. Show the rhombus in the rectangle. Discover the base and height. Rename these the long and short diagonals.





5. Use the extra pink pieces to see that the rectangle is made up of 2 rhombi.

6. Write the formula: $A = \frac{D\,d}{2}$

D =long diagonal
d = short diagonal

Follow up: Task cards for area of rhombus.

Area of a Kite

Direct Aim: Development of a formula for the area of a kite.

Indirect Aim: Application for real life area problems.

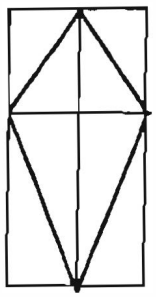
Prerequisites: Area of the rectangle.

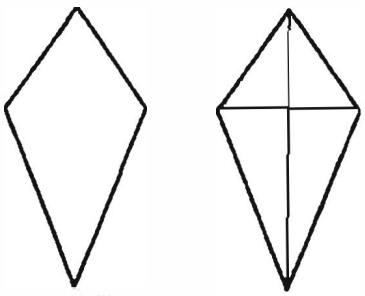
Materials:
- Package of kite area material
- Geometry notebook

Presentation:

1. Agree that a kite is somewhat like a rhombus. Can a base or a height be found? Show the kite from the area material.

2. Can a long diagonal and a short diagonal be found? Show the material that has the lines marked.





3. Use the pieces to form two kites. Develop the formula.

$$A = \frac{Dd}{2}$$

Area of a Trapezoid

Direct Aim: Development of a formula for the area of a trapezoid.

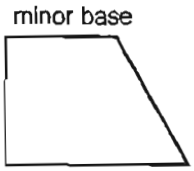
Indirect Aim: Real life work with the area of a trapezoid.

Prerequisites: Area of the triangle.

Materials: • Package of trapezoid area material
• Geometry notebook

Presentation:

1. Take the trapezoid out of the package. Discuss how you were able to explore and change the trapezoid into a rectangle.



2. Recall taking the minor base and turning it upside down to extend the base, making a longer rectangle.

major base

3: Show the trapezoid with the minor base extended and the triangular piece cut off.

4. Examine the resulting triangle. Its base is equal to the major base plus the minor base. Its height is equal to the height of the trapezoid.

5. The formula for the area of a trapezoid must be the same as the formula for the area of a triangle. Thus we have: $A = \frac{(B+b)h}{2}$

B = major base
b = minor base

Follow up: Trapezoid area task cards.

Area of an Irregular Quadrilateral

Direct Aim: Development of a formula for the area of an irregular quadrilateral.

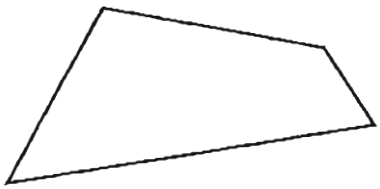
Indirect Aim: Ability to find the area of any quadrilateral.

Prerequisites: Area of the triangle.

Materials: • Package of irregular quadrilateral area material
• Geometry notebook

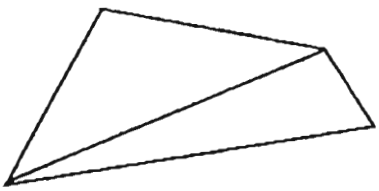
Presentation:

1. Examine an irregular quadrilateral. Search for a height and a base.



2. Since no height or base can be distinguished, the area will be found by using the triangle as a mediator.

3. Draw a line dividing the quadrilateral diagonally. Examine the resulting triangles.



4. Find the area of each of the two triangles. $A_Q = A_a + A_b$

5. Student records formula in geometry notebook.

Follow Up: Geoboards, Creative Publications Task Cards.

Area of a Polygon

Direct Aim: Development of a formula for the area of a polygon.

Indirect Aim: Ability to solve real life polygon area problems.

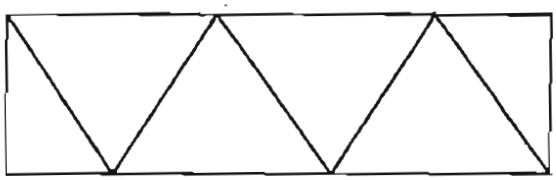
Prerequisites: Area of the rectangle.

Materials: • Package of polygon area material
• Geometry notebook

Presentation:

1. Take the pentagon out of the package. Examine and discuss possible ways to discover the area.

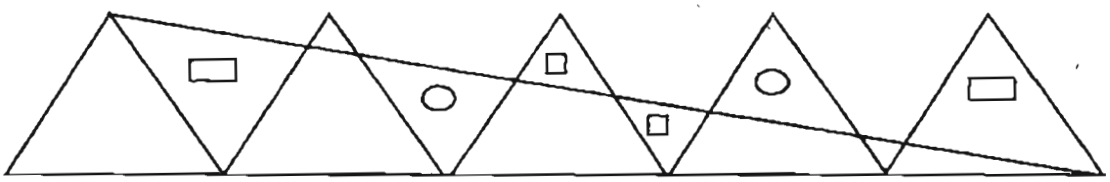
2. When the students suggest cutting into triangles, take these out and attempt build a rectangle. Examine the lines in the rectangle. Where do they come from in the polygon?



3. We can develop the formula : $A = \frac{1}{2}Pa$

(Area = one half Perimeter times apothem)

3. We can also develop another formula. Show line of triangles. Notice that the perimeter of the polygon is now the base of the line of triangles. The apothem of the polygon is the height of the triangles.



Surface Area of a Sphere

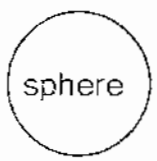
Prerequisites: The area of a circle.

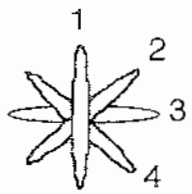
Materials:

- Sphere
- Frame from largest circle of Geometric cabinet
- Packet of Volume of sphere material
- Geometry notebook

Presentation:

1. Pass the frame over the sphere. Note that it passes in any direction. Pass over horizontally
   vertically
   two obliques.

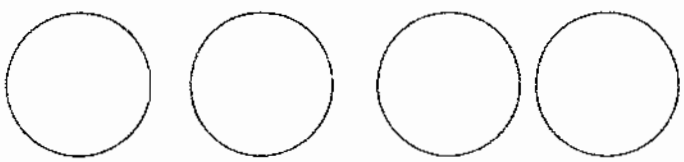




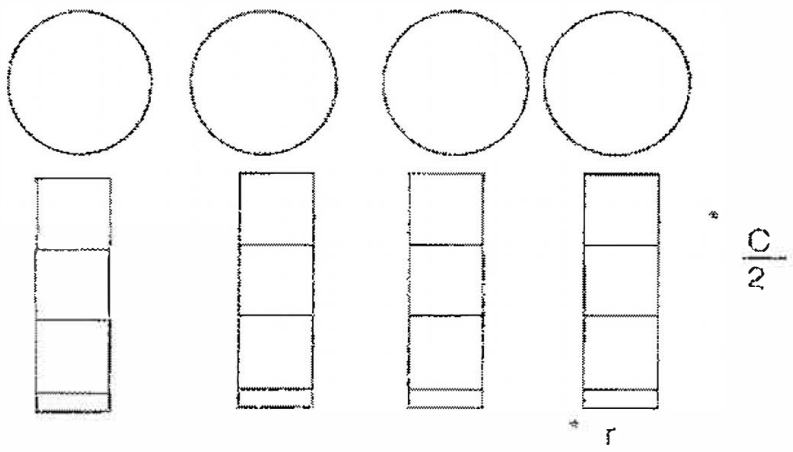
2. We have made the "Four Great Circles" named by Archimedes.

3. These "Four Great Circles", according to Archimedes equal the surface area of the sphere. This was discovered.by placing a hollow leaf of iron on one pan and four great circles of the same material on the other pan. They balanced.

4. Lay the four circles from the Volume packet out on the rug.



5. Recall that each circle makes a rectangle. Lay out the corresponding rectangle.



6. Recognize each rectangle as being πr^2. Label

7. Rearrange the rectangles into a long rectangle.

Now we see $4\pi r^2$.

8. The spherical surface area is $4\pi r^2$

* We remember that the base of each small special rectangle is equal to half of the circumference of the circle and the height of the rectangle is equal to the radius of the circle.

Notes

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