## UNDERSTANDING \& USING

DECIMALS


Set A

## Set A (Introduction to Decimals) - Scope and Sequence

| Card \# | Concepts | Operations | Drill |
| :---: | :---: | :---: | :---: |
| 1 | Naming whole numbers | 6-9 Hierarchy review | Writing place values |
| 2 | Naming decimal numbers | Intro to decimals lesson <br> *See teacher instructions if <br> necessary | Compare values |
| 3 | Naming decimals on a <br> hierarchy board |  <br> words | Writing in digits |
| 4 | Changing decimals to <br> mixed fractions | Candelabra | Recognizing place value |
| 5 | Changing fractions to <br> decimals | Recognizing operations <br> and nomenclature (add) | Layout from pictures |

## Set A Instructions

## Introduction to Decimals card 2

We commonly use the term "decimal" to mean "decimal fraction," a value less than one.

Using a piece of green paper, draw the largest possible circle. This will represent 1 unit. Cut it out.

From the diameter, use a protractor to measure $36^{\circ}$. Cut the ten resulting sectors. These represent tenths. Glue $1 / 10$ onto a sheet of yellow paper and label it with both the fractional $\left(\frac{1}{10}\right)$ and decimal ( 0.1 ) values. We are building a decimal fraction hierarchy table.

Take one of the "tenths" and cut it into 10 pieces. These represent hundredths. Glue one of the $1 / 100$ 's to the yellow paper, to the right of the tenth. Label it with both the fractional and decimal values.

Continue dividing into smaller units until you can no longer cut pieces or you reach 1/1,000,000.

Compare your hierarchy table to the hierarchy board. Note that each piece you made was 10 times smaller than the previous piece. Note that on the hierarchy board the color of the value changes, but not the size. Why do you think the color changes?

## Introduction to Decimals card 4

Bring the decimal hierarchy board to your space. Examine it carefully. Notice that the unit is the most important number because it is the base from which the hierarchies, both larger and smaller, extend.

Using a piece of graph paper we will draw our number system hierarchy.
Color in a green square in the center of your page. Turn the page sideways and write in "units."

On the board, notice the color of the hierarchies on the two sides of the unit. On your page, skip one square on each side, then color in the next square. Turn your page sideways and write the value of the place.


Continue until you have colored and named all the place values from the board.

Since the unit is the most important number, draw a line from the green square to the bottom of your page. Draw in a triangular
base. Add in arms from the line that comes off the unit to each of the place values. What does your drawing resemble?

## Introduction to Decimals card 5

ONE is the most important number! If you don't have one, you can't count.
We can show that one is the midpoint of all the hierarchies by making a pinwheel.
You will need a piece of graph paper, some cardstock, green, blue, and red colored pencils, scissors, a pushpin and a pencil with an eraser.


In the center of the graph paper outline a square. Write " 1 " inside the square. On the row above, write "10", with the " 1 " in the next column, and the " 0 " above the original unit, 1.

Continue each time moving up and out by one row and column until you reach the million's place.

Next move down and out, using decimal fractions. Be sure to put "0" in the unit's place, a decimal point between the boxes, then " 1 " in the tenth's, and so on to the millionth's place.

Now color the columns according to their place values. Cut out the pinwheel and glue it onto cardstock. Cut out the cardstock and place a pushpin in the unit's square. Using the pushpin, attach the pinwheel to the eraser of a pencil. Blow. See how the unit is critical?

Set A Answers

|  | Concepts | Operations | Drills |
| :---: | :---: | :---: | :---: |
| 1 | - Forty-five thousand, six hundred twenty-eight <br> - One hundred twenty-six million, three hundred five thousand, six hundred forty-one | Drawing of material with place value nomenclature | 136,006 <br> 3,341,322 <br> Two hundred ninety-three million, six hundred fifty-five thousand, four hundred four |
| 2 | - Twenty-six and two hundred eightyfour thousandths <br> - Eighteen and two thousand seven hundred eighty-six ten thousandths | Introduction to decimals lesson | $\begin{aligned} & 24,420,833 \\ & 126,720 \\ & \text { August } \end{aligned}$ |
| 3 | - Sixteen and thirty-eight hundredths <br> - Seven and nine thousand three hundred seventy-six ten thousandths | Candelabra lesson | $\begin{array}{\|l\|} \hline 189 \\ 15,012,504 \\ 510,072,000 \end{array}$ |
| 4 | - $\frac{35}{100}$ <br> - $7 \frac{289}{1000}$ <br> - $25 \frac{8}{10}$ <br> - $475 \frac{3976}{10000}$ | Layout on hierarchy board | - 6,266 <br> - 1440 <br> - four 10 millions or forty millions |
| 5 | - 54.607 <br> - .912 <br> - .24 <br> - 6.9 | Pinwheel lesson | $\begin{aligned} & \hline 31,109 \\ & 185 \\ & 162 \text { seats } \end{aligned}$ |
| 6 | - 62,395 \& 3,586 addends; 65,981 sum <br> - $12,845 \& 6,736$ addends; 19,581 sum | - 1.023 .101242 <br> - . 1034337.07035 | $\begin{array}{\|lll} \hline \text { a) } 251 & \text { b) } 207 & \text { c) } 267 \\ 596,925 \mathrm{~km}^{2} & \\ \hline \end{array}$ |
| 7 | - 32,694 minuend; 6475 subtrahend; 26,219 difference <br> - 3,958 minuend; 1,587 subtrahend; 2,371 difference | - Bead layout | $\begin{array}{\|l\|} \hline 1,310 \\ 14,994 \\ 8 \text { colors } \end{array}$ |
| 8 | - 54 multiplicand \& 8 multiplier; 432 product <br> - 83 multiplicand \& 16 multiplier; 1,328 product <br> - 34,257 \& 734 factors; $25,144,638$ product | - . 75405.350401 <br> - 60.40244 .5575 | $\begin{array}{\|l\|} \hline 6,524 \\ 11,508 \\ \text { They both ate } 3 \text { pieces. } \end{array}$ |
| 9 | - 8 divisor, 5096 dividend, 637 quotient | - Bead layout | $\begin{array}{\|l\|} \hline 382 \\ 4,328 \\ 126 \text { rabbits } \\ \hline \end{array}$ |
| 10 | - 15 multiplicand \& 5 multiplier, 75 product <br> - 12,465 dividend; 15 divisor; 831 quotient <br> - 5,796 minuend; 2,388 subtrahend; 3,408 difference | - 2.07045 .3607 <br> - . 30501635.046 | a) 428 b) 486 c) 459 .39 is better because: $.39 \times 3=\$ 1.17$ |
| 11 | $\begin{array}{ll} \hline \bullet & 5,740 \\ \bullet & 65,460 \\ \bullet & 3,840 \\ \hline \end{array}$ | - Bead layout | $\begin{array}{\|l\|} \hline 168 \\ 33,327,240 \\ \text { Apples }=\$ 4.95 \\ \hline \end{array}$ |
| 12 | - 10,000 | - . 057171470.611 | 360,232 |


|  | - 8,600 <br> -900  <br> - 2,000 | - 10.61 12.5201 | $\begin{aligned} & \hline 683 \text { r } 71 \\ & 203 \text { miles } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| 13 | - $10^{\text {th }}=2.4,45.6,12.4$ <br> - 100 ths $=12.58,8.85,34.39$ | - Challenge | $\begin{aligned} & \hline 194 \text { r } 31 \\ & 23,664 \\ & 3 \text { white, } 4 \text { red } \\ & \hline \end{aligned}$ |
| 14 | - $10^{\text {th }}=2.438 .6 \quad 6.7$ <br> - $100^{\text {th }}=2.3938 .596 .75$ | - $45.27 \quad 1023.0332$ <br> - 4.051063 .246 | $\begin{aligned} & \hline 4,800 \\ & 1,500 \\ & 950 \mathrm{~m}(1,700 \mathrm{ft}) \\ & \hline \end{aligned}$ |
| 15 | $\begin{array}{ll}\bullet & 1000^{\text {th }}=3.460 \quad 93.584 \quad 2.741 \\ \bullet & 10,000^{\text {th }}=3.4597 \quad 93.5836 \\ 2.7409\end{array}$ | - Challenge | $\$ 5$ $\$ 46$ $\$ 7$ 3 volunteers |

## Set B

Outline for Set B (Adding \& Subtracting Decimals) - Scope \& Sequence

| Card \# | Concepts | Operations | Drill |
| :---: | :---: | :---: | :---: |
| 1 | Decimals to fractions (no <br> reducing) | Adding decimals | Naming decimals and fractions |
| 2 | Decimals to fractions <br> (reducing) | Adding decimals | Measurement comparisons |
| 3 | Decimal in tenths to <br> fractions | Adding decimals | Naming decimals in words |
| 4 | Decimal in hundredths to <br> fraction | Adding decimals | Sequencing decimals and |
| fractions |  |  |  |

## Teacher Instructions Set B

## Addition of decimals

Review with the student the nomenclature for addition, and the rules for decimals:
Keep the decimal lined up when you add.
Rewrite all problems vertically.
NOTE: Students rarely have difficulty with addition of decimals.

## Card 6: Subtraction of decimals

Review with the student the nomenclature for subtraction, and the rules for decimals:
Keep the decimal lined up when you subtract.
Rewrite all problems vertically.
Remind the student that in decimal numbers a smaller number value can have more digits than a larger value. Examine this concept with the student.

## $4.5>4.2374$

When subtracting it is IMPERATIVE to line up the decimal point. Any digits in the subtrahend that have no corresponding digits in the minuend will require dynamic subtraction, i.e. exchanging.

Set B Answers

| Cd | Concepts | Operations | Drills |
| :---: | :---: | :---: | :---: |
| 1 | $\begin{array}{lll}\text { - } & \frac{346}{1000} & 2 \frac{93}{100} \\ \text { - } & \frac{9452}{10000} & 6 \frac{3}{10}\end{array}$ | - 238.078 <br> - 975.9106 <br> - 200.00007 | - 36.9 <br> - 3.658 <br> - .25 or $\frac{1}{4}$ |
| 2 | $\begin{array}{ll} \hline \text { - } \frac{4}{5} \\ \text { - } \frac{3}{4} \end{array}$ | $\begin{array}{ll}-\quad 638.199 \\ - & 40.0092 \\ -\quad 100.2798\end{array}$ | $\begin{array}{ll}\text { - } & 313 \text { r } 30 \\ \text { - } & 5,280 \text { feet }\end{array}$ <br> - October |
| 3 | - $\frac{3}{5}$ $\frac{1}{2}$ | $\begin{array}{ll}- & 11.6101 \\ \bullet & 104.7746 \\ - & 104.4767\end{array}$ | - Forty-six thousand twentyseven and six hundred three thousandths <br> - Three hundred twentyeight and seven hundred four ten thousandths <br> - 6 books |
| 4 | - $\frac{1}{4} \frac{1}{5}$ <br> - $\frac{13}{20} \frac{8}{25}$ | - 114.0713 <br> - 12.7208 <br> - 39.1014 | - 6.079, 6.709, 6.79, 6.97 <br> - 16,021 <br> - 5 volunteers |
| 5 | $\begin{array}{llr}\text { - } & \frac{1}{8} & \frac{11}{20} \\ \text { - } & \frac{27}{125} & \frac{5}{8}\end{array}$ | - 11.4302 <br> - 143.4468 <br> - 10.6816 | - a) 302 b) 352 c) 322 <br> - 20 packages |
| 6 | Kinds of averages; definitions | - 1.559 <br> - 34.3212 | - 3337 r 6 |
| 7 | - Organized list of salaries | - 44.31 <br> - 20.806 <br> - 50.376 | - 696.686 <br> - 79.5036 <br> - 539 miles |
| 8 | - Anthony Beltran, \$105,500 | - 189.613 <br> - 63.088 <br> - 5.463 | - 1569 r 11 <br> - 36.75 <br> - 12.27 <br> - Answers will vary |
| 9 | $\begin{array}{r} \bullet \quad 25 \\ \bullet \quad 26 \end{array}$ | - <br> $-\quad 4.134$ <br> $-\quad 9.239$ |  |
| 10 | Answers will vary | $\begin{array}{ll}\bullet & 7.442 \\ \bullet & 11.453 \\ - & 4.326\end{array}$ | - 504.678, 504.768, 504.786 <br> - 31 games |
| 11 | Answers will vary | $\begin{array}{ll}- & 22.166 \\ \bullet & 62.253 \\ - & 114.53\end{array}$ | - 2031 r 8 <br> - \$10,996,399 |
| 12 | $-\quad 22$ $-\quad 25$ | - 97.0178 <br> - 5.915 <br> - 12.226 | - 17,360 633.61 <br> - 18 choices |
| 13 | Answers will vary | - 153.1134 <br> - 7.646 <br> - 4.603 | - 20.48 .02 <br> - 2 rows |
| 14 | Answers will vary | - 23.5966 <br> - 143.4853 <br> - 2.90964 | - 800045,000 <br> - 5 hours |


| 15 | - 11 cm <br> - 17 pages | $\begin{array}{ll} \hline- & 12.974 \\ - & 60.7606 \\ - & 382.8023 \\ \hline \end{array}$ | - 60313.0113 <br> - $\quad 140 \mathrm{ft}$ |
| :---: | :---: | :---: | :---: |

## Set C

## Outline for Set C (Multiplying Decimals) - Scope \& Sequence

| Card \# | Concepts | Operations | Drill |
| :---: | :---: | :---: | :---: |
| 1 | Comparing using < > | $X$ Dec by whole | Rounding decimal numbers |
| 2 | Comparing using <> | X Dec by whole | Mean |
| 3 | Comparing using < > | Hierarcy card game lesson | Estimates |
| 4 | Comparing using < > | Hierarcy card game | Decimals to fractions |
| 5 | Translating to math shorthand | X Dec by whole | Average |
| 6 | Translating; variables | Into checkerboard | Inequalities |
| 7 | Translating | $X$ Dec by dec (1 digit) | Rounding decimals |
| 8 | Translating | $X$ Dec by dec (1 digit) | Mode |
| 9 | Translating | $X$ Dec by dec (1 digit) | Inequalities |
| 10 | Translating | $X$ Dec by dec (1 digit) | Decimals to fractions |
| 11 | Multiplicative identity | $X$ Dec by dec (2 digit) | Rounding |
| 12 | Commutative property | $X$ Dec by dec (2 digit) | Algebraic nomenclature |
| 13 | Associative property | X Dec by dec (2 digit) | Estimates |
| 14 | Distributive property | $X$ Dec by dec (2 digit) | Decimal to fraction |
| 15 | Mixed properties | $X$ Dec by dec (3 digit) | Median |
| 16 | Exponents intro | $X$ Dec by dec | Distributive property |
| 17 | Power of 1 rule | X Dec by dec | Algebraic nomenclature |
| 18 | Zero rule | $X$ Dec by dec | Estimates |
| 19 | Powers of ten | $X$ Dec by dec | Mean |
| 20 | Exponential notation | X Dec by dec | Exponents |
| 21 | Standard notation | X Dec by dec | Inequalities |
| 22 | Product rule | $X$ Dec by dec | Associative property |
| 23 | Power rule | X Dec by dec | Exponents |
| 24 | Quotient rule | $X$ Dec by dec | Mode |
| 25 | Practice | X Dec by dec | Scientific notation |

## Teacher Instructions for Set C (Multiplication of Decimals)

As students begin multiplying decimals, it is extremely important to understand the place values. When multiplying decimals times whole numbers, it is very simple, since multiplication is repeated addition and the math facts hold true.

## Lesson on Place Value Card Game

On the following page is a set of six 10 cm squares. Two copies of the page need to be made in the following colors: red, blue, green, light red (or pink), light blue, and light green. (Or use 2 sheets of each color cardstock and cut 10 cm squares) The darker colors will represent whole numbers, the lighter ones, decimal numbers.

Place one unit card in the center of the table; mix all the other cards. Hand out the same number of cards to each player. Players stack the cards in front of them. Place the extras to the side as the "go fish" pile.

Each player has to play the top card on his stack. If he can't, he may "go fish" until he has a card he can play.
To play, the player must take the top card off his stack and place it next to any card on the table, either horizontally or vertically and name it all the way back to the nearest unit. For example : This is a hundredth. Hundredths, tenths, units!"

Variations:

1) Do not place a unit to start. Begin with the first player setting down a card, naming it and others having to "grow" from that place value.
2) Allow empty place values by leaving "holes"

## Introduction to Decimal Checkerboard

Compare the checkerboard to the decimal card game. See how the colors are lighter on one side and darker on the other? What does this represent? Why? Look at the numbers written along the frame. What do they represent?

When students start multiplying decimals times decimals they should notice that multiplied numbers become SMALLER rather than bigger. This is because decimals are fractions. Every student should understand that $8 \times 1 / 2$ is 4 . So they can follow that $8 \times$ 0.5 also equals 4 , because $.5=1 / 2$. Prove this on the checkerboard.

## NOTE:

1) As students layout on the board it is imperative that they ALWAYS place two tiles in the unit's place.
2) Student MUST vocalize each operation as they complete it:
"Tenths times tenths is hundredths"

## Exponent Rules:

Product rule: When multiplying two powers that have the same base, you can add the exponents.

Zero as an exponent always gives you one.
Power rule: To raise a power to a power, just multiply the exponents.

In Set D we continue with exponents to learn:
Quotient rule: When we divide two powers with the same base we simply subtract the exponents.
Negative exponents make decimal numbers.



Set C Answers

| Cd | Concepts | Operations | Drill |
| :---: | :---: | :---: | :---: |
| 1 | $\begin{aligned} & \bullet> \\ & \bullet \\ & \bullet \end{aligned}$ | - 1.778 <br> - 2.4335 | - 38.60 <br> - 29.58 <br> - 100.00 <br> - 24 different numbers |
| 2 | - greater than, less than <br> - less than, greater than | - 1.7067 <br> - 7.808 <br> - 5.74 | - 86 r 29 <br> - 92,584 <br> - $\quad 109.75 \mathrm{lbs}$ (diff of 2.25 ) |
| 3 | - $123<132$ <br> - $14.56>14.506$ <br> - $2.625>21 / 2$ <br> - $0.3<0.35$ | Hierarchy card game | - 107.4155 <br> - 2.3852 <br> - $\approx \$ 5.00$ |
| 4 | - $345>34.5$ <br> - $3.66<36.6$ <br> - $2,478>247.8$ <br> - $.501<.510$ | Hierarchy card game | - = <br> - $\neq\left(2^{3 / 4}\right)$ <br> - = <br> - 40 students |
| 5 | - $>$ greater $\geq$ greater or equal to <br> - $\neq$ not equal $=$ equal <br> - < less $\leq$ less or equal to | $\begin{array}{ll} \hline \bullet & 30.576 \\ \bullet & 27.39 \\ \bullet & 27.864 \\ \hline \end{array}$ | - 52 <br> - 27 <br> - 7 pallets |
| 6 | $\begin{array}{ll} \text { - } & x-6=11 \\ - & 42=7 x \\ - & x+8=12 \\ \hline \end{array}$ | - 1.038 <br> - 2.94 <br> - 7.047 | - = <br> - > <br> - $3: 30 \mathrm{pm}$ |
| 7 | - $492.6>491.9$ <br> - $x+5 \leq 10$ <br> - $x=2(6)$ | - $\quad 1.4535$ <br> - 6.545 | $\begin{array}{lll} \hline \bullet & 345.6 & 345.63 \\ \bullet & 4.9 & 4.90 \\ \bullet & 9.3 & 9.27 \\ \bullet & 84 \text { days } \\ \hline \end{array}$ |
| 8 | - Five times six equals thirty <br> - 248 taken 36 times equals 8928 <br> - 12 multiplied by 87 is 1044 <br> - One half of 364 is 182 | $\begin{array}{ll} \hline \bullet & 3.7912 \\ \bullet & 1.45 \\ \bullet & 8.1567 \end{array}$ | - 48 <br> - 351,918 <br> - median: 7.75; mean: 7.6; mode: $8^{1 / 2}$ |
| 9 | - $14=x-11$ <br> - $56 \div 28=x$ <br> - $4 x+2(1)=\$ 6.87$ | $-\quad 0.2112$ $-\quad 1.9675$ $-\quad 2.2734$ | - $n<7$ <br> - $x>22$ <br> - 50 days |
| 10 | - $\quad P=3 J$ <br> - $S=2 B$ <br> - $R=1 / 2 S$ | - <br>  <br> -$\quad 0.88432$ | - $23 / 4,9^{3 / 5}, 3^{14} / 25$ <br> - 24 leaves |
| 11 | Any number added to zero is the number itself. Any number multiplied by 1 is the number itself. | - 0.29601 <br> - 1.3593 <br> - 13.6404 | - 84.3984 .386 <br> - .75 .754 <br> - $16.98 \quad 16,984$ <br> - 19 cases, 4 leftovers |
| 12 | - 10.9 <br> - 28.48 <br> - 16.37 | - 1.001 <br> - 12.4614 <br> - 0.031598 | - $\quad \mathrm{S}+(\mathrm{S}+3)=15, \quad 27=\mathrm{T}-\mathrm{J}$ <br> $M=1 / 2(166)$ <br> - 15 cans |
| 13 | - 16.067 | - 6.25818 <br> - 2.5536 <br> - 3.79144 | - 15,000, $80,000,21,000$ <br> - Heidi $=7$; Grandad $=56$ |
| 14 | - $32+48=80$ <br> - $32+7.0=39$ | - 3.0096 <br> - 2.83936 <br> - 20.2358 | - $\mathrm{n}>19, \mathrm{x}<8.75$ <br> - Toni - baseball - doctor <br> - Destiny - tennis beautician <br> - Bethany - dancing - teacher |


| 15 | $\begin{array}{\|lll} \hline \bullet & \mathrm{AP} \\ \bullet & \mathrm{DP} \\ \bullet & \mathrm{IP} \\ \bullet & \mathrm{CP} \\ \bullet & \mathrm{CP} \\ \bullet & \mathrm{IP} \end{array}$ | - 16.7188 <br> - 2.57735 <br> - 35.33153 | - 25 (in 2009) |
| :---: | :---: | :---: | :---: |
| 16 | $\begin{array}{\|ll} \bullet- & 216 \\ \bullet & 81 \\ \bullet & 49 \end{array}$ | - 23.94942 <br> - 34.2126 <br> - 83.6182 | - $8+10=18 ; 32-12=20$; $42+35=77$ <br> - 8 people. 2 boys and 2 girls, their mother and father, the mother's mother and the father's father. (remember a family member can be both a sister and a daughter) |
| 17 | $\begin{array}{\|ll} \bullet & 512 \\ \bullet & 144 \\ \bullet & 46 \\ \bullet & 25 \\ \bullet & 4 \end{array}$ | - 8.56976 <br> - 45.9359 <br> - 8.66272 | - 20 <br> - 44 <br> - $1591+4=1595$ |
| 18 | - $5^{0}=1$ <br> - $7^{0}=1$ <br> - any \# to 0 power = 1 | $\begin{array}{ll}\bullet & 7.10288 \\ \bullet & 9.36514 \\ - & 2.878282\end{array}$ | - about 20 cans about 70 / crate <br> - map |
| 19 | - $10=1$ <br> - $10_{2}=10$ <br> - $10_{3}^{2}=100$ <br> - $10=1000$ <br> - continue | - 7.79625 <br> - 5.185275 <br> - 246.39564 | - 27.8 years old (in 2009) |
| 20 | - $\left(10^{3} \times 3\right)+\left(10^{2} \times 4\right)+\left(10^{1} \times 8\right)+\left(10^{0} \times 5\right)$ <br> - $(10 \times 2)+(10 \times 8)+(10 \times 3)+(10 \times 7)$ | $\begin{array}{ll}\bullet & 33.282 \\ \bullet & 35.2836 \\ - & 0.196558\end{array}$ | - $645,13,136$ <br> - about $\$ 35$ |
| 21 | - 57,364 - 9,931 $-50,653$ | - 5.0416 <br> - 15.7076 <br> - 2.63088 | - $n>6$ <br> - $x<9$ <br> - 85 comic books |
| 22 | - $8^{2} \cdot 8^{5}=8^{7}$ <br> - $6^{3} \cdot 6^{3}=6^{6}$ <br> - $5^{2} \cdot 5^{3}=5^{5}$ <br> - Add the exponents | - 0.020634 <br> - 6.8179 <br> - 278.184 | - $(5 \times 12) x=60 x$ <br> - $(24 \cdot 4) 8=576$ <br> - 4 socks |
| 23 | - Sun $149,597,887 \mathrm{~km}$ <br> - Moon 384,483 km <br> - Varies | - 1.09011 <br> - $\quad 37.961$ <br> - | - $4^{5} \cdot 5^{3} ; 8^{4} \cdot 4^{4} \cdot 6^{6} ; 2^{5} \cdot 6^{4}$ <br> - 24 possibilities |
| 24 | - Sun $1.4 \times 10^{8} \mathrm{~km}$ <br> - Moon $3.8 \times 10^{5} \mathrm{~km}$ <br> - Varies | - $\quad 1.30005$ $-\quad 28.6433$ $-\quad 47.0376$ | - 24 years old (in 2009) |
| 25 | - $6.7 \times 10^{9}$ miles/hour <br> - $1.4 \times 10^{9}$ years old <br> - $1.0 \times 10^{14}$ cells in our bodies | - 11.2233 <br> - 314.3136 <br> - 46.875 <br> - the trick works because they are decimal FRACTIONS | - $\quad 2.3 \times 10^{14}$ <br> - $\quad 5.4 \times 10^{9}$ <br> - $6.4 \times 10^{8}$ <br> - Sandra $=10 ;$ Marcus $=2$ |

## Set D

## Outline for Set D (Division of Decimals)

| Cd | Concepts | Operations | Drill |
| :---: | :---: | :---: | :---: |
| 1 | Exponent to a power | 1 digit whole \# divisor | Standard/expanded notation |
| 2 | Dividing exponents | 1 digit whole \# divisor | Average |
| 3 | Negative exponents | 2 digit whole \# divisor | Estimating products |
| 4 | Order of operations | 2 digit whole \# divisor | Order of operations |
| 5 | Order of operations | 2 digit whole \# divisor | Multiplication of decimal \& inequalities |
| 6 | Fraction to decimal | 2 digit whole \#. divisor | Median |
| 7 | Fraction to decimal | Decimal $\div$ lesson | Exponents |
| 8 | Divisibility 2 | 1 digit decimal divisor | Rounding |
| 9 | Divisibility 5 | 1 digit decimal divisor | Order of operations (+, - ) |
| 10 | Divisibility 10 | 2 digit decimal divisor | Negative exponents |
| 11 | Divisibility 3 | 2 digit decimal divisor | Order of operations |
| 12 | Divisibility 4 | 2 digit decimal divisor | Fraction to decimal |
| 13 | Practice divisibility | 3 digit decimal divisor | Estimating sums |
| 14 | Divisibility 6 | 2 digit decimal divisor | Mode |
| 15 | Divisibility 9 | 3 digit decimal divisor | Order of operations |
| 16 | Practice divisibility | 2 digit decimal divisor | Comparing values (frac \& dec) |
| 17 | Divisibility 8 | 3 digit decimal divisor | Expanded notation w/ negative exponents |
| 18 | Introduce percents | 3 digit decimal divisor | Inequalities |
| 19 | \% Equivalencies | 3 digit decimal divisor | Average (mean) |
| 20 | Percent formula | 2 digit decimal divisor | Percent to fraction \& decimal |
| 21 | Finding percentage | 3 digit decimal divisor | Expanded notation |
| 22 | Finding percentage | 3 digit decimal divisor | Find the percentage |
| 23 | Finding percentage | 3 digit decimal divisor | Percent to fraction to decimal |
| 24 | Finding rate | 3 digit decimal divisor | Median |
| 25 | Finding base | 3 digit decimal divisor | Find the rate |

## Teacher Instructions for Set D

## Division of a decimal by a whole number

Dividing a decimal by a whole number is similar to dividing money, because the decimal point is fixed. However there are differences.

1) The decimal point doesn't have to be in the hundredth's place (as it is with money).
2) Zeros are added after the last digit, until either the quotient ends, or repeats. (Or you may choose to have your students stop after the 10,000ths place)

## Division of a decimal by a decimal number

Explain that dividing by a decimal is like taking a skittle, dividing it into 10 pieces, then using a portion of those pieces to divide. We could be using tiny pieces! So instead we change the whole problem into a fraction.
First, ask the student, which is the numerator and which is the denominator?
Agree that $. 8 \longdiv { 2 . 6 1 2 }$ is the same as -
The first step is to turn the denominator into a whole number. We use the multiplicative identity to decide:

-     - so the corresponding problem becomes - - -

This problem can be changed back into the "normal" division format and divided.
There are 7 steps to be shown in the work:

1) Write the problem
2) Rewrite it as a fraction
3) Find the multiplicative identity that will make the denominator whole
4) Rewrite the new fraction
5) Move the decimal point in the division problem using carets
6) Divide
7) Check the answer using the ORIGINAL divisor as the multiplier.

## Percentage

Changing fractions to percents and to decimals is easy to do because you just need to have a denominator of 100 for everything to work out. When the fraction cannot be changed with the multiplicative identity to have a denominator of 100 , simply divide the numerator by the denominator to get a decimal. When it is in the hundredths place, eliminate the decimal point and add a percent sign.

## Percent Formula

Begin by stating the definitions of the three parts of the formula:
Base = foundation or what you start out with
Rate $=$ ratio $=$ fraction or percent (a fraction with a denominator of 100)
Percentage $=$ a portion of the base or the result of 'percenting'

Note that percentage is often used wrongly as people think it is the number with the \% sign. But that is really the rate! To help students understand use this parallel:

Pack is an action that you take. When you've 'packed', the result is a 'package'.
Percent is an action that you take. When you've 'percented' the result is a 'percentage'.

Point out that the $P$ in the formula ( $P=b r$ ) is always a capital letter, while the $b$ and $r$ are always lower case. Use other formulas to show that the result is always a capital while the factors are lower case. ( $\mathrm{A}=\mathrm{bh}$ )

Ask the questions to set up the formula:
$\mathrm{P}=$
$\mathrm{b}=$
$r=$
What are you trying to find?
What version of the formula should you use?

Using the definitions answer each of the first three questions. These will lead to the answers of the last two.

Answers Set D

| Cardd | Concepts | Operations | Drill |
| :---: | :---: | :---: | :---: |
| 1 | $\begin{aligned} & \bullet \\ & \bullet \\ & \bullet \\ & -2^{12}+3^{4} \\ & \hline \end{aligned}$ | $\begin{array}{ll}- & 15.4 \\ \bullet & 0.026 \\ - & 0.085\end{array}$ | - 64,806 <br> - 8,373 <br> - $3 \times 10^{3}+5 \times 10^{2}+8 \times 10^{1}+4 \times 10^{0}$ <br> - $2 \times 10^{6}+3 \times 10^{5}+8 \times 10^{4}+5 \times 10^{3}+9 \times 10^{2}+$ $4 \times 10^{1}+6 \times 10^{0}$ <br> - (N) Angel (clockwise) Peter, Destiny, Simon |
| 2 | - $3^{1}$ <br> - $4^{3}$ <br> - $7^{1}$ <br> - Exponents are subtracted | - 1.08 <br> - 0.123 <br> - 0.54 | - 140, 209, 369 <br> - 180 cuckoos in 24 hours |
| 3 | - $\frac{1}{8^{3}} \frac{1}{10^{8}} \frac{1}{12^{2}}$ <br> - $\quad 0.0001,0.000001$, | - 0.45 <br> - 21.6 <br> - 1.388 | - $250,000,120,000,490,000$ <br> - arrive home at 12:01 am |
| 4 | - 110 <br> - 83.4 | - 0.08 <br> - 0.925 <br> - 0.038 | - 263,440 <br> - 34,$064 ; 0 ; 4825.000001$ <br> - 3 hot dogs \& 2 buns |
| 5 | - 69.4 <br> - $28,882.6$ <br> - 907 | $\begin{array}{ll}- & 0.655 \\ \bullet & 0.3575 \\ - & 0.026\end{array}$ | - $11.27>10.07$ <br> - $2.189>1.889$ <br> - $5.4352>5.3928$ <br> - 14 minutes |
| 6 | - $0.3 \overline{3}$ <br> - 0.125 <br> - $0.83 \overline{3}$ <br> - 0.7142857 <br> - $0.1 \overline{1}$ | - 0.805 <br> - 0.365 <br> - 4.315 | - 6 satellites per country |
| 7 | $\bullet$ 0.375 <br> $\bullet$ 0.5714285 <br> $\bullet$ $0.1 \overline{6}$ <br> $\bullet$ 0.8 <br> - $0.5 \overline{5}$ | - 3.256 <br> - 1.755 | - 2,352; 2,187; 4,096; 4,096; 134 <br> - 108 DVDs |
| 8 | - all even numbers | $\begin{aligned} & \bullet \cdot \\ & \bullet \\ & \bullet \end{aligned} .4755$ | - 0.2065; 53.5069; 27.335 <br> - $2\left(40^{\text {th }}\right.$ and $80^{\text {th }}$ box) |
| 9 | - all ending in 5 or 0 | - .35 <br> - 8.04875 <br> - 223.75 | - $6+2-4-2-1=1$ <br> - $3+6-2+5-2=10$ <br> - $4+2+4-2-2=6$ <br> - 17 hair pieces |
| 10 | - See Appendix D | $\begin{array}{ll}\text { - } & 2.06 \\ - & 1.82 \\ - & 18.54\end{array}$ | - . 00053; 2.0306; . 0000018 <br> - $\$ 75$ with 5 and $\$ 327.67$ with 1 © |
| 11 | - 21, 69, 27, 93, 78, <br> 645, 432, 702, 72 | $\begin{array}{ll} \bullet & .55 \\ \bullet & 3.4 \\ \bullet & 24.5 \\ \hline \end{array}$ | - $51,49,3$ <br> - 16 horses \& 20 people |
| 12 | - 248, 576, 524, 796, | - 8.62 | - 9.5; .777; 6.2; 5.916666; 125 |


|  | 780,676 | - $\quad 6.025$ $-\quad 8.316$ | - leave home at 1:13 |
| :---: | :---: | :---: | :---: |
| 13 | - See appendix A | - 3.50340625 <br> - . 365 <br> - 3.862 | - $150,310,210$ <br> - $54 i n, 48 i n, 44 i n$ |
| 14 | - 24, 66, 936, 282, 510 | - 201.21428 <br> - 2.2230 r 0.0002 <br> - 1040.2222 r 0.000002 | - mode $=4$ <br> - Philip, Precious, Victoria, Sarah, Michael |
| 15 | $\begin{aligned} & \bullet \quad 81,207,567,801, \\ & 333,108,531,243, \\ & 297,8,748 \end{aligned}$ | - 387 <br> - 13.9576 r 0.000168 <br> - 49.2 | - $3+2-1+3-6=1$ <br> - $8-3+2+5-11=1$ <br> - $7+4-6+3-7=1$ <br> - 600 marbles |
| 16 | See appendix B | - 11.9388 r0.00032 <br> - 18.2 <br> - 4.2 | - . 08; . 12; . 125 (1/8); . 42 ; . $6(3 / 5)$ <br> - about 22 or 23 years |
| 17 | - 3,808; 5,624; <br> 6,552; 2,536; 8,512 | - 1.5673 r 0.000016 <br> - 2.4365 r 0.00035 <br> - 1.2959 r 0.00007 | - $3 \times 10^{2}+6 \times 10^{1}+7 \times 10^{0}+9 \times 10^{-1}+3 \times 10^{-2}$ <br> - $2 \times 10^{0}+3 \times 10^{-1}+9 \times 10^{-2}+7 \times 10^{-4}$ <br> - $6 \times 10^{1}+4 \times 10^{0}+3 \times 10^{-3}$ <br> - 27 choices |
| 18 | - $20 \%, 10 \%, 75 \%$ <br> - $3 / 10,21 / 25,1 / 20$ | - 214.6956 r 0.0000024 <br> - 21.3268 r 0.00006 <br> - 1340 | - $14<16$ <br> - 159.42 > 144.52 <br> - 74 pts |
| 19 | - $331 / 3 \%, 40 \%$, $371 / 2 \%, 2 / 3,1 / 8$ | - $\quad 0.3017$ r 0.000196 <br> - 14.7123 r 0.000484 <br> - 0.8745 r 0.00015 | - 39.75 Ohio <br> - 56.75 New York <br> - 106.75 Michigan <br> - 134.25 Alaska <br> - $1,371 \mathrm{CDs}$ |
| 20 | - $\mathrm{P}=96$ so 224 customers to finish | $\begin{array}{ll}\bullet & 63.8360 \text { r } 0.000004 \\ - & 533.1944 \text { r } 0.0000032\end{array}$ <br> - 4.9763 r 0.000035 | - $\quad 0.62 ; 0.44 ; 0.3 ; 0.78 ; 0.24$ <br> - 57 pennies |
| 21 | 2,800 seats sold, 700 seats left | - $1953.3333 r$ <br> 0.0000016 <br> - 105.31944 <br> - 1.9093 r 0.000028 | - $1 \times 10^{4}+2 \times 10^{3}+8 \times 10^{2}+4 \times 10^{1}+6 \times 10^{0}$ <br> - $2 \times 10^{2}+1 \times 10^{1}+4 \times 10^{0}+4 \times 10^{-2}+5 \times 10^{-3}$ <br> - $3 \times 10^{1}+2 \times 10^{0}+8 \times 10^{-3}+9 \times 10^{-4}$ <br> - $3 \times 10^{3}+4 \times 10^{2}+8 \times 10^{1}+5 \times 10^{0}+2 \times 10^{-1}$ <br> - $8 \times 10^{2}+7 \times 10^{1}+3 \times 10^{0}+6 \times 10^{-1}$ <br> - 8 and 5 |
| 22 | - \$25 off, pay $\$ 100$ | - . 5101952 <br> - 1.4641 r 0.00036 <br> - 5.4875 r 0.00025 | - 10.56; 23.92; 36.48; 3 <br> - Natalie 22, Olivia 11, Selena 33, Georgina 31 |
| 23 | - 26 more $=243$ potatoes | - 39.651376 - - 3.6548 | - $.79,{ }^{79} / 100$ <br> - $.84,{ }_{3}^{21} / 25$ <br> - $.12, \frac{3}{2} 25$ <br> - $.55,{ }^{11} / 20$ <br> - $.36,9 / 25$ <br> - $\$ 340$ |
| 24 | - $53 \%$ | - 22.9945 r 0.00012 <br> - 3.0609 r 0.000093 <br> - 59.55 | - 37.1 inches of rain per year |
| 25 | - 568 persimmons | - 18.0357 r 0.0000044 <br> - 312.5666 r 0.000002 <br> - 0.6746 r 0.000124 | - $50 \%, 46 \%, 75 \%, 80 \%$ <br> - Left, 9 trees, left |

Appendix A

| Divisible by | 2 | 4 | 5 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 12 | ( $\sqrt{ }$ ) | $(\sqrt{ })$ | ( ) | ( ) |
| 15 | ( ) | ( ) | ( $\sqrt{ }$ ) | ( ) |
| 25 | ( ) | ( ) | $(\sqrt{ })$ | ( ) |
| 60 | $(\sqrt{ }$ ) | $(\sqrt{ })$ | ( $\sqrt{ }$ ) | ( $\sqrt{ }$ ) |
| 84 | $(\sqrt{ }$ ) | $(\sqrt{ }$ ) | ( ) | ( ) |
| 220 | ( $\sqrt{ }$ ) | $(\sqrt{ })$ | ( $\sqrt{ }$ ) | ( $\sqrt{ }$ ) |
| 715 | ( ) | ( ) | $(\sqrt{ })$ | ( ) |

Appendix B

| Divisible by | 3 | 6 | 9 |
| :---: | :---: | :---: | :---: |
| 1,920 | ( $\sqrt{ }$ ) | $(\sqrt{ })$ | ( ) |
| 5,614 | ( ) | ( ) | ( ) |
| 4,713 | ( $\sqrt{ }$ ) | ( ) | ( ) |
| 5,040 | ( ) | ( ) | ( $\sqrt{ }$ ) |
| 6,003 | $(\sqrt{ })$ | ( ) | $(\sqrt{ })$ |
| 1,368 | (V) | $(\sqrt{ })$ | ( $\sqrt{ }$ ) |
| 7,458 | $(\sqrt{ }$ ) | $(\sqrt{ })$ | ( ) |
| 3,645 | (V) | ( ) | ( $\sqrt{ }$ ) |
| 7,683 | ( $\sqrt{ }$ ) | ( ) | ( ) |

Appendix D

| Divisible by | 2 | 5 | 10 |
| :--- | :--- | :--- | :--- |
| 6 | $(\sqrt{ })$ | () | $(~)$ |
| 42 | $(\sqrt{ })$ | $(~)$ | $(~)$ |
| 634 | $(\sqrt{ })$ | () | () |
| 205 | () | $(\sqrt{ })$ | () |
| 830 | $(\sqrt{ })$ | $(\sqrt{ })$ | $(\sqrt{ })$ |
| 1,405 | () | $(\sqrt{ })$ | () |
| 3,700 | $(\sqrt{ })$ | $(\sqrt{ })$ | $(\sqrt{ })$ |

