UNDERSTANDING & USING DECIMALS



ANALYSIS SYNTHESIS READING WRITING

NUMERATION PATTERNS OPERATIONS



Set A

Set A (Introduction to Decimals) – Scope and Sequence	
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Card #	Concepts	Operations	Drill
1	Naming whole numbers	6-9 Hierarchy review	Writing place values
2	Naming decimal numbers	Intro to decimals lesson *See teacher instructions if necessary	Compare values
3	Naming decimals on a hierarchy board	Layout from fractions & words	Writing in digits
4	Changing decimals to mixed fractions	Candelabra	Recognizing place value
5	Changing fractions to decimals	Pinwheel	Multiplication
6	Recognizing operations and nomenclature (add)	Layout from pictures	Adding
7	Recognizing operations and nomenclature (sub.)	Layout from words & fractions	Draw a picture to add fractions
8	Recognizing operations and nomenclature (mult.)	Layout from pictures	Draw a picture to subtract fractions
9	Recognizing operations and nomenclature (divide)	Layout from words	Make a table to find a pattern
10	More division	Layout from pictures	Multiply and compare numbers
11	Recognizing operations	Layout from words	Multiply
12	Rounding whole numbers	Layout from pictures	Multiply
13	"Estimating"	Challenge	Make a table with LCM
14	Rounding Decimals	Challenge	Estimating products
15	Rounding Decimals	Challenge	Estimating money

Set A Instructions

Introduction to Decimals card 2

We commonly use the term "decimal" to mean "decimal fraction," a value less than one.

Using a piece of green paper, draw the largest possible circle. This will represent 1 unit. Cut it out.

From the diameter, use a protractor to measure 36° . Cut the ten resulting sectors. These represent tenths. Glue 1/10 onto a sheet of yellow paper and label it with both the fractional $(\frac{1}{10})$ and decimal (0.1) values. We are building a decimal fraction hierarchy table.

Take one of the "tenths" and cut it into 10 pieces. These represent hundredths. Glue one of the 1/100's to the yellow paper, to the right of the tenth. Label it with both the fractional and decimal values.

Continue dividing into smaller units until you can no longer cut pieces or you reach 1/1,000,000.

Compare your hierarchy table to the hierarchy board. Note that each piece you made was 10 times smaller than the previous piece. Note that on the hierarchy board the color of the value changes, but not the size. Why do you think the color changes?

Introduction to Decimals card 4

Bring the decimal hierarchy board to your space. Examine it carefully. Notice that the unit is the most important number because it is the base from which the hierarchies, both larger and smaller, extend.

Using a piece of graph paper we will draw our number system hierarchy.

Color in a green square in the center of your page. Turn the page sideways and write in "units."

On the board, notice the color of the hierarchies on the two sides of the unit. On your page, skip one square on each side, then color in the next square. Turn your page sideways and write the value of the place.



Continue until you have colored and named all the place values from the board.

Since the unit is the most important number, draw a line from the green square to the bottom of your page. Draw in a triangular

base. Add in arms from the line that comes off the unit to each of the place values. What does your drawing resemble?

Introduction to Decimals card 5

ONE is the most important number! If you don't have one, you can't count.

We can show that one is the midpoint of all the hierarchies by making a pinwheel.

You will need a piece of graph paper, some cardstock, green, blue, and red colored pencils, scissors, a pushpin and a pencil with an eraser.



In the center of the graph paper outline a square. Write "1" inside the square. On the row above, write "10", with the "1" in the next column, and the "0" above the original unit, 1.

Continue each time moving up and out by one row and column until you reach the million's place.

Next move down and out, using decimal fractions. Be sure to put "0" in the unit's place, a decimal point between the boxes, then "1" in the tenth's, and so on to the millionth's place.

Now color the columns according to their place values. Cut out the pinwheel and glue it onto cardstock. Cut out the cardstock and place a pushpin in the unit's square. Using the pushpin, attach the pinwheel to the eraser of a pencil. Blow. See how the unit is critical?

Set A Answers

	Concepts	Operations	Drills
1	 Forty-five thousand, six hundred twenty-eight One hundred twenty-six million, three hundred five thousand, six hundred forty-one Twenty-six and two hundred eighty- four thousandths 	Drawing of material with place value nomenclature Introduction to decimals	136,006 3,341,322 Two hundred ninety-three million, six hundred fifty-five thousand, four hundred four 24,420,833 126,720
	Eighteen and two thousand seven hundred eighty-six ten thousandths	lesson	August
3	 Sixteen and thirty-eight hundredths Seven and nine thousand three hundred seventy-six ten thousandths 	Candelabra lesson	189 15,012,504 510,072,000
4	• $\frac{35}{100}$ • $7\frac{289}{1000}$ • $25\frac{8}{10}$ • $475\frac{3976}{10000}$	Layout on hierarchy board	 6,266 1440 four 10 millions or forty millions
5	 54.607 .912 .24 6.9 	Pinwheel lesson	31,109 185 162 seats
6	 62,395 & 3,586 addends; 65,981 sum 12,845 & 6,736 addends; 19,581 sum 	 1.023 .101242 .1034 337.07035 	a) 251 b) 207 c) 267 596,925 km²
7	 32,694 minuend; 6475 subtrahend; 26,219 difference 3,958 minuend; 1,587 subtrahend; 2,371 difference 	Bead layout	1,310 14,994 8 colors
8	 54 multiplicand & 8 multiplier; 432 product 83 multiplicand & 16 multiplier; 1,328 product 34,257 & 734 factors; 25,144,638 product 	 .75405 .350401 60.402 44.5575 	6,524 11,508 They both ate 3 pieces.
9	 8 divisor, 5096 dividend, 637 quotient 	Bead layout	382 4,328 126 rabbits
10	 15 multiplicand & 5 multiplier, 75 product 12,465 dividend; 15 divisor; 831 quotient 5,796 minuend; 2,388 subtrahend; 3,408 difference 	 2.0704 5.3607 .305016 35.046 	a) 428 b) 486 c) 459 .39 is better because: .39 x 3 = \$1.17
11	 5,740 65,460 3,840 	Bead layout	168 33,327,240 Apples = \$4.95
12	• 10,000	• .057171 470.611	360,232

	8,6009002,000	• 10.61 12.5201	683 r 71 203 miles
13	 10th = 2.4, 45.6, 12.4 100ths = 12.58, 8.85, 34.39 	Challenge	194 r 31 23,664 3 white, 4 red
14	• $10^{\text{th}} = 2.4$ 38.6 6.7 • $100^{\text{th}} = 2.39$ 38.59 6.75	 45.27 1023.0332 4.05106 3.246 	4,800 1,500 950 m (1,700 ft)
15	 1000th = 3.460 93.584 2.741 10,000th = 3.4597 93.5836 2.7409 	Challenge	\$5 \$46 \$7 3 volunteers

Set B

Outline for Set B (Adding & Subtracting Decimals) – Scope & Sequence

Card #	Concepts	Operations	Drill
1	Decimals to fractions (no reducing)	Adding decimals	Naming decimals and fractions
2	Decimals to fractions (reducing)	Adding decimals	Measurement comparisons
3	Decimal in tenths to fractions	Adding decimals	Naming decimals in words
4	Decimal in hundredths to fraction	Adding decimals	Sequencing decimals and fractions
5	Decimal in thousandths to fraction	Adding decimals	Estimation
6	Estimating	Subtracting decimals	Decimals to fractions
7	Organizing soccer salary list for median	Subtracting decimals	Addition of decimals
8	Finding the median	Subtracting decimals	Rounding to hundredths
9	Calculating median	Subtracting decimals	Decimal to fraction
10	Collecting data for mean	Subtracting decimals	Sequencing decimals
11	Finding the mean of collected data	Mixed addition and subtraction	Arithmetic mean
12	Calculating mean	Mixed addition and subtraction	Draw a picture
13	Mode	Mixed addition and subtraction	Fraction to decimal
14	When to use median, mean, mode	Mixed addition and subtraction	Estimation
15	Using mean	Mixed addition and subtraction	Decimal subtraction

Teacher Instructions Set B

Addition of decimals

Review with the student the nomenclature for addition, and the rules for decimals: Keep the decimal lined up when you add. Rewrite all problems vertically.

NOTE: Students rarely have difficulty with addition of decimals.

Card 6: Subtraction of decimals

Review with the student the nomenclature for subtraction, and the rules for decimals: Keep the decimal lined up when you subtract. Rewrite all problems vertically.

Remind the student that in decimal numbers a smaller number value can have more digits than a larger value. Examine this concept with the student.

4.5 > 4.2374

When subtracting it is IMPERATIVE to line up the decimal point. Any digits in the subtrahend that have no corresponding digits in the minuend will require dynamic subtraction, i.e. exchanging.

Set B Answers

Cd	Concepts	Operations	Drills
1	• $\frac{346}{1000}$ 2 $\frac{93}{100}$ • $\frac{9452}{10000}$ 6 $\frac{3}{10}$	 238.078 975.9106 200.00007 	 36.9 3.658 .25 or ¹/₄
2	$ \begin{array}{c} \bullet \frac{4}{5} \\ \bullet \frac{3}{4} \end{array} $	638.19940.0092100.2798	 313 r 30 5,280 feet October
3	• $\frac{3}{5}$ $\frac{1}{2}$ • $\frac{1}{5}$ $\frac{2}{5}$	 11.6101 104.7746 104.4767 	 Forty-six thousand twenty-seven and six hundred three thousandths Three hundred twenty-eight and seven hundred four ten thousandths 6 books
4	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	 114.0713 12.7208 39.1014 	 6.079, 6.709, 6.79, 6.97 16,021 5 volunteers
5	• $\frac{1}{8}$ $\frac{11}{20}$ • $\frac{27}{125}$ $\frac{5}{8}$	 11.4302 143.4468 10.6816 	 a) 302 b) 352 c) 322 20 packages
6	Kinds of averages; definitions	1.55934.3212	 3337 r 6 ³⁄₄ , 1½, 12 1/8 29 bracelets
7	Organized list of salaries	 44.31 20.806 50.376 	 696.686 79.5036 539 miles
8	 Anthony Beltran, \$105,500 	 189.613 63.088 5.463 	 1569r11 36.75 12.27 Answers will vary
9	• 25 • 26	 2.134 4.609 9.239 	• $3\frac{\frac{86}{100}}{97\frac{46}{10000}}$ • 8 points
10	Answers will vary	 7.442 11.453 4.326 	 504.678, 504.768, 504.786 31 games
11	Answers will vary	 22.166 62.253 114.53 	 2031 r 8 \$10,996,399
12	• 22 • 25	 97.0178 5.915 12.226 	 17,360 633.61 18 choices
13	Answers will vary	 153.1134 7.646 4.603 	20.4 8.02 2 rows
14	Answers will vary	 23.5966 143.4853 2.90964 	 8000 45,000 5 hours

4.5	• 11cm	٠	12.974	٠	603 13.0113	
15		•	60.7606	•	140 ft	
	• 17 pages	•	382.8023			

Set C

Outline for Set C (Multiplying Decimals) – Scope & Sequence

Card #	Concepts	Operations	Drill
1	Comparing using < >	X Dec by whole	Rounding decimal numbers
2	Comparing using < >	X Dec by whole	Mean
3	Comparing using < >	Hierarcy card game lesson	Estimates
4	Comparing using < >	Hierarcy card game	Decimals to fractions
5	Translating to math shorthand	X Dec by whole	Average
6	Translating; variables	Into checkerboard	Inequalities
7	Translating	X Dec by dec (1 digit)	Rounding decimals
8	Translating	X Dec by dec (1 digit)	Mode
9	Translating	X Dec by dec (1 digit)	Inequalities
10	Translating	X Dec by dec (1 digit)	Decimals to fractions
11	Multiplicative identity	X Dec by dec (2 digit)	Rounding
12	Commutative property	X Dec by dec (2 digit)	Algebraic nomenclature
13	Associative property	X Dec by dec (2 digit)	Estimates
14	Distributive property	X Dec by dec (2 digit)	Decimal to fraction
15	Mixed properties	X Dec by dec (3 digit)	Median
16	Exponents intro	X Dec by dec	Distributive property
17	Power of 1 rule	X Dec by dec	Algebraic nomenclature
18	Zero rule	X Dec by dec	Estimates
19	Powers of ten	X Dec by dec	Mean
20	Exponential notation	X Dec by dec	Exponents
21	Standard notation	X Dec by dec	Inequalities
22	Product rule	X Dec by dec	Associative property
23	Power rule	X Dec by dec	Exponents
24	Quotient rule	X Dec by dec	Mode
25	Practice	X Dec by dec	Scientific notation

Teacher Instructions for Set C (Multiplication of Decimals)

As students begin multiplying decimals, it is extremely important to understand the place values. When multiplying decimals times whole numbers, it is very simple, since multiplication is repeated addition and the math facts hold true.

Lesson on Place Value Card Game

On the following page is a set of six 10cm squares. Two copies of the page need to be made in the following colors: red, blue, green, light red (or pink), light blue, and light green. (Or use 2 sheets of each color cardstock and cut 10 cm squares) The darker colors will represent whole numbers, the lighter ones, decimal numbers.

Place one unit card in the center of the table; mix all the other cards. Hand out the same number of cards to each player. Players stack the cards in front of them. Place the extras to the side as the "go fish" pile.

Each player has to play the top card on his stack. If he can't, he may "go fish" until he has a card he can play.

To play, the player must take the top card off his stack and place it next to any card on the table, either **horizontally** or **vertically** and name it all the way back to the nearest unit. For example : This is a hundredth. Hundredths, tenths, units!" Variations:

- 1) Do not place a unit to start. Begin with the first player setting down a card, naming it and others having to "grow" from that place value.
- 2) Allow empty place values by leaving "holes"

Introduction to Decimal Checkerboard

Compare the checkerboard to the decimal card game. See how the colors are lighter on one side and darker on the other? What does this represent? Why? Look at the numbers written along the frame. What do they represent?

When students start multiplying decimals times decimals they should notice that multiplied numbers become SMALLER rather than bigger. This is because decimals are fractions. Every student should understand that 8 x $\frac{1}{2}$ is 4. So they can follow that 8 x 0.5 also equals 4, because .5 = $\frac{1}{2}$. Prove this on the checkerboard.

NOTE:

1) As students layout on the board it is imperative that they **ALWAYS** place two tiles in the unit's place.

2) Student **MUST** vocalize each operation as they complete it:

"Tenths times tenths is hundredths"

Exponent Rules:

Product rule: When multiplying two powers that have the same base, you can add the exponents.

Zero as an exponent always gives you one.

Power rule: To raise a power to a power, just multiply the exponents.

In Set D we continue with exponents to learn:

Quotient rule: When we divide two powers with the same base we simply subtract the exponents.

Negative exponents make decimal numbers.





Cd	Concepts	Operations	Drill
1	• > < • > <	1.7782.4335	 38.60 29.58 100.00 24 different numbers
2	greater than, less thanless than, greater than	1.70677.8085.74	 86 r 29 92,584 109.75 lbs (diff of 2.25)
3	 123 < 132 14.56 > 14.506 2.625 > 2 ½ 0.3 < 0.35 	Hierarchy card game	 107.4155 2.3852 ≈ \$5.00
4	 345 > 34.5 3.66 < 36.6 2,478 > 247.8 .501 < .510 	Hierarchy card game	 = ≠ (2 ³/₄) = 40 students
5	 > greater ≥ greater or equal to ≠ not equal = equal < less ≤ less or equal to 	30.57627.3927.864	 52 27 7 pallets
6	 x - 6 = 11 42 = 7x x + 8 = 12 	1.0382.947.047	 = > 3:30 pm
7	 492.6 > 491.9 x +5 ≤ 10 x = 2(6) 	1.45350.22686.545	 345.6 345.63 4.9 4.90 9.3 9.27 84 days
8	 Five times six equals thirty 248 taken 36 times equals 8928 12 multiplied by 87 is 1044 One half of 364 is 182 	 3.7912 1.45 8.1567 	 48 351,918 median: 7.75; mean: 7.6; mode: 8½
9	 14 = x -11 56 ÷ 28 = x 4x + 2(1) = \$6.87 	 0.2112 1.9675 2.2734 	 <i>n</i><7 <i>x</i>> 22 50 days
10	 P = 3J S = 2B R = ½ S 	2.88425.99320.5472	 2 ³⁄₄, 9 ³/₅, 3 ¹⁴/₂₅ 24 leaves
11	Any number added to zero is the number itself. Any number multiplied by 1 is the number itself.	0.296011.359313.6404	 84.39 84.386 .75 .754 16.98 16,984 19 cases, 4 leftovers
12	 10.9 28.48 16.37 	1.00112.46140.031598	 S+(S+3)=15, 27=T-J M= ½ (166) 15 cans
13	• 16.067	6.258182.55363.79144	 15,000, 80,000, 21,000 Heidi = 7; Grandad = 56
14	 32 + 48 = 80 32 + 7.0 = 39 	 3.0096 2.83936 20.2358 	 n >19, x < 8.75 Toni – baseball – doctor Destiny – tennis – beautician Bethany – dancing - teacher

15	 AP DP IP CP CP IP 	 16.7188 2.57735 35.33153 	• 25 (in 2009)
16	 216 81 49 	 23.94942 34.2126 83.6182 	 8+10 =18; 32 - 12 = 20; 42 + 35 = 77 8 people. 2 boys and 2 girls, their mother and father, the mother's mother and the father's father. (remember a family member can be both a sister and a daughter)
17	 512 144 46 25 4 	 8.56976 45.9359 8.66272 	 20 44 1591 + 4 = 1595
18	 5⁰ = 1 7⁰ = 1 any # to 0 power = 1 	 7.10288 9.36514 2.878282 	 about 20 cans about 70 / crate map
19	• $10^{\circ} = 1$ • $10^{\circ} = 10$ • $10^{\circ} = 100$ • $10^{\circ} = 1000$ • continue	 7.79625 5.185275 246.39564 	• 27.8 years old (in 2009)
20	• $(10^{3}x3)+(10^{2}x4)+(10^{1}x8)+(10^{0}x5)$ • $(10^{3}x2)+(10^{2}x8)+(10^{1}x3)+(10^{0}x7)$	 33.282 35.2836 0.196558 	645, 13, 136about \$35
21	 57,364 9,931 50,653 	5.041615.70762.63088	 n > 6 x < 9 85 comic books
22	• $8^2 \cdot 8^5 = 8^7$ • $6^3 \cdot 6^3 = 6^6$ • $5^2 \cdot 5^3 = 5^5$ • Add the exponents	 0.020634 6.8179 278.184 	 (5 x 12)x = 60x (24•4)8 = 576 4 socks
23	 Sun 149,597,887 km Moon 384,483 km Varies 	1.0901137.961221.892	 4⁵ • 5³; 8⁴ • 4⁴ • 6⁶; 2⁵ • 6⁴ 24 possibilities
24	 Sun 1.4 x 10⁸ km Moon 3.8 x 10⁵ km Varies 	1.3000528.643347.0376	• 24 years old (in 2009)
25	 6.7 x 10⁹ miles/hour 1.4 x 10⁹ years old 1.0 x 10¹⁴ cells in our bodies 	 11.2233 314.3136 46.875 the trick works because they are decimal FRACTIONS 	• 2.3×10^{14} • 5.4×10^{9} • 6.4×10^{8} • Sandra = 10; Marcus = 2

Set D

Cd	Concepts	Operations	Drill
1	Exponent to a power	1 digit whole # divisor	Standard/expanded notation
2	Dividing exponents	1 digit whole # divisor	Average
3	Negative exponents	2 digit whole # divisor	Estimating products
4	Order of operations	2 digit whole # divisor	Order of operations
5	Order of operations	2 digit whole # divisor	Multiplication of decimal & inequalities
6	Fraction to decimal	2 digit whole #. divisor	Median
7	Fraction to decimal	Decimal ÷ lesson	Exponents
8	Divisibility 2	1 digit decimal divisor	Rounding
9	Divisibility 5	1 digit decimal divisor	Order of operations (+, -)
10	Divisibility 10	2 digit decimal divisor	Negative exponents
11	Divisibility 3	2 digit decimal divisor	Order of operations
12	Divisibility 4	2 digit decimal divisor	Fraction to decimal
13	Practice divisibility	3 digit decimal divisor	Estimating sums
14	Divisibility 6	2 digit decimal divisor	Mode
15	Divisibility 9	3 digit decimal divisor	Order of operations
16	Practice divisibility	2 digit decimal divisor	Comparing values (frac & dec)
17	Divisibility 8	3 digit decimal divisor	Expanded notation w/ negative exponents
18	Introduce percents	3 digit decimal divisor	Inequalities
19	% Equivalencies	3 digit decimal divisor	Average (mean)
20	Percent formula	2 digit decimal divisor	Percent to fraction & decimal
21	Finding percentage	3 digit decimal divisor	Expanded notation
22	Finding percentage	3 digit decimal divisor	Find the percentage
23	Finding percentage	3 digit decimal divisor	Percent to fraction to decimal
24	Finding rate	3 digit decimal divisor	Median
25	Finding base	3 digit decimal divisor	Find the rate

Outline for Set D (Division of Decimals)

Teacher Instructions for Set D

Division of a decimal by a whole number

Dividing a decimal by a whole number is similar to dividing money, because the decimal point is fixed. However there are differences.

- 1) The decimal point doesn't have to be in the hundredth's place (as it is with money).
- 2) Zeros are added after the last digit, until either the quotient ends, or repeats.(Or you may choose to have your students stop after the 10,000ths place)

Division of a decimal by a decimal number

Explain that dividing by a decimal is like taking a skittle, dividing it into 10 pieces, then using a portion of those pieces to divide. We could be using tiny pieces! So instead we change the whole problem into a fraction.

First, ask the student, which is the numerator and which is the denominator?

Agree that .8)2.612 is the same as —

The first step is to turn the denominator into a whole number. We use the multiplicative identity to decide:

— — so the corresponding problem becomes — — —

This problem can be changed back into the "normal" division format and divided. There are 7 steps to be shown in the work:

- 1) Write the problem
- 2) Rewrite it as a fraction
- 3) Find the multiplicative identity that will make the denominator whole
- 4) Rewrite the new fraction
- 5) Move the decimal point in the division problem using carets
- 6) Divide
- 7) Check the answer using the ORIGINAL divisor as the multiplier.

Percentage

Changing fractions to percents and to decimals is easy to do because you just need to have a denominator of 100 for everything to work out. When the fraction cannot be changed with the multiplicative identity to have a denominator of 100, simply divide the numerator by the denominator to get a decimal. When it is in the hundredths place, eliminate the decimal point and add a percent sign.

Percent Formula

Begin by stating the definitions of the three parts of the formula: Base = foundation or what you start out with Rate = ratio = fraction or percent (a fraction with a denominator of 100) Percentage = a portion of the base or the result of 'percenting'

Note that percentage is often used wrongly as people think it is the number with the % sign. But that is really the rate! To help students understand use this parallel: Pack is an action that you take. When you've 'packed', the result is a 'package'. Percent is an action that you take. When you've 'percented' the result is a 'percentage'.

Point out that the P in the formula (P=br) is always a capital letter, while the b and r are always lower case. Use other formulas to show that the result is always a capital while the factors are lower case. (A=bh)

Ask the questions to set up the formula:

P =

b =

r =

What are you trying to find?

What version of the formula should you use?

Using the definitions answer each of the first three questions. These will lead to the answers of the last two.

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Cardd	Concepts	Operations	Drill
1	• $6^{12} + 3^4$ • $2^5 + 3^4$	 15.4 0.026 0.085 	 64,806 8,373 3 x 10³ + 5 x 10² + 8 x 10¹ + 4 x 10⁰ 2x10⁶+3x10⁵+8x10⁴+5x10³+9x10²+ 4x10¹+6x10⁰ (N) Angel (clockwise) Peter, Destiny, Simon
2	 3¹ 4³ 7¹ Exponents are subtracted 	 1.08 0.123 0.54 	 140, 209, 369 180 cuckoos in 24 hours
3	• $\frac{1}{8^3} \frac{1}{10^8} \frac{1}{12^2}$ • 0.0001, 0.000001, 0.000001, 0.001	0.4521.61.388	 250,000, 120,000, 490,000 arrive home at 12:01 am
4	11083.4	 0.08 0.925 0.038 	 263,440 34,064; 0; 4825.000001 3 hot dogs & 2 buns
5	 69.4 28,882.6 907 	 0.655 0.3575 0.026 	 11.27 > 10.07 2.189 > 1.889 5.4352 > 5.3928 14 minutes
6	 0.33 0.125 0.833 0.7142857 0.11 	 0.805 0.365 4.315 	6 satellites per country
7	 0.375 0.5714285 0.16 0.8 0.55 	3.2561.755	 2,352; 2,187; 4,096; 4,096; 134 108 DVDs
8	all even numbers	 .525 .475 	 0.2065; 53.5069; 27.335 2 (40th and 80th box)
9	• all ending in 5 or 0	 .35 8.04875 223.75 	• $6+2-4-2-1=1$ • $3+6-2+5-2=10$ • $4+2+4-2-2=6$ • 17 hair pieces
10	See Appendix D	 2.06 1.82 18.54 	 .00053; 2.0306; .0000018 \$75 with 5 and \$327.67 with 1¢
11	• 21, 69, 27, 93, 78, 645, 432, 702, 72	 .55 3.4 24.5 	 51, 49, 3 16 horses & 20 people
12	 248, 576, 524, 796, 616, 936, 348, 64, 	• 8.62	• 9.5; .777; 6.2; 5.916666; .125

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	780, 676	 6.025 8.316 	leave home at 1:13
13	• See appendix A	 3.50340625 .365 3.862 	 150, 310, 210 54in, 48in, 44in
14	• 24, 66, 936, 282, 510	 201.21428 2.2230 r 0.0002 1040.2222 r 0.000002 	 mode = 4 Philip, Precious, Victoria, Sarah, Michael
15	 81, 207, 567, 801, 333, 108, 531, 243, 297, 8,748 	 387 13.9576 r 0.000168 49.2 	• $3+2-1+3-6=1$ • $8-3+2+5-11=1$ • $7+4-6+3-7=1$ • 600 marbles
16	See appendix B	 11.9388 r0.00032 18.2 4.2 	 .08; .12; .125 (1/8); .42; .6 (3/5) about 22 or 23 years
17	 3,808; 5,624; 6,552; 2,536; 8,512 	 1.5673 r 0.000016 2.4365 r 0.00035 1.2959 r 0.00007 	• $3x10^{2}+6x10^{1}+7x10^{0}+9x10^{-1}+3x10^{-2}$ • $2x10^{0}+3x10^{-1}+9x10^{-2}+7x10^{-4}$ • $6x10^{1}+4x10^{0}+3x10^{-3}$ • 27 choices
18	 20%, 10%, 75% ³/₁₀, ²¹/₂₅, ¹/₂₀ 	 214.6956 r 0.0000024 21.3268 r 0.00006 1340 	 14 < 16 159.42 > 144.52 74 pts
19	 ³³¹/₃%, 40%, 37¹/₂%, ²/₃, ¹/₈ 	 0.3017 r 0.000196 14.7123 r 0.000484 0.8745 r 0.00015 	 39.75 Ohio 56.75 New York 106.75 Michigan 134.25 Alaska 1,371 CDs
20	P=96 so 224 customers to finish	 63.8360 r 0.000004 533.1944 r 0.0000032 4.9763 r 0.000035 	 0.62; 0.44; 0.3; 0.78; 0.24 57 pennies
21	2,800 seats sold, 700 seats left	 1953.3333 r 0.0000016 105.31944 1.9093 r 0.000028 	• $1 \times 10^{4} + 2 \times 10^{3} + 8 \times 10^{2} + 4 \times 10^{1} + 6 \times 10^{0}$ • $2 \times 10^{2} + 1 \times 10^{1} + 4 \times 10^{0} + 4 \times 10^{-2} + 5 \times 10^{-3}$ • $3 \times 10^{1} + 2 \times 10^{0} + 8 \times 10^{-3} + 9 \times 10^{-4}$ • $3 \times 10^{3} + 4 \times 10^{2} + 8 \times 10^{1} + 5 \times 10^{0} + 2 \times 10^{-1}$ • $8 \times 10^{2} + 7 \times 10^{1} + 3 \times 10^{0} + 6 \times 10^{-1}$ • $8 \text{ and } 5$
22	• \$25 off, pay \$100	 .5101952 1.4641 r 0.00036 5.4875 r 0.00025 	 10.56; 23.92; 36.48; 3 Natalie 22, Olivia 11, Selena 33, Georgina 31
23	• 26 more= 243 potatoes	 39.651376 3.6548 	• $.79, \frac{79}{100}$ • $.84, \frac{21}{25}$ • $.12, \frac{3}{25}$ • $.55, \frac{11}{20}$ • $.36, \frac{9}{25}$ • $$340$
24	• 53%	 22.9945 r 0.00012 3.0609 r 0.000093 59.55 	• 37.1 inches of rain per year
25	568 persimmons	 18.0357 r 0.0000044 312.5666 r 0.000002 0.6746 r 0.000124 	 50%, 46%, 75%, 80% Left, 9 trees, left

Appendix A

Divisible by	2	4	5	10
12 15 25 60 84 220 715	() () () ($)$ ($$) ($$) ()	() ()) ($)$ ($$) ($$) ($$)	() (√) (√) () () () ()	() () (√) (√) (√)

Appendix B

Divisible by	3	6	9
1,920 5,614 4,713 5,040 6,003 1,368 7,458 3,645 7,683	(\checkmark) (\checkmark) (\checkmark) (\checkmark) (\checkmark) (\checkmark) (\checkmark) (\checkmark)	(√) () () () () (√) ()	()))()()()()()()()()()()()()()()()()()
Appendix D			
Divisible by	2	5	10
6	(√)	()	()
42	(√)	()	()
634	(√)	()	()
205	()	(√)	()
830	(√)	(√)	(√)
1,405	()	(√)	()
3,700	(√)	(√)	(√)